Optimizing Process Economic Performance with Model Based Control

James B. Rawlings

Department of Chemical and Biological Engineering

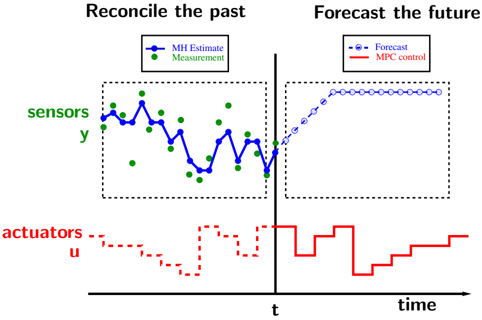
THE UNIVERSITY of WISCONSIN MADISON

May 5, 2011
Model Based Control Conference
Danish Technical University

Outline

1. Optimal control, optimal feedback control, and model predictive control (MPC)
2. Industrial impact of these ideas
3. Using MPC to optimize plant economics
4. Summary Remarks

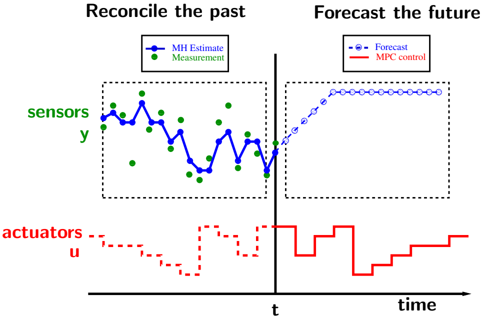
Predictive control



$$\min_{u(t)} \int_0^T |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2 \, dt$$

$$\dot{x} = f(x,u)$$
$$x(0) = x_0 \quad \text{(given)}$$
$$y = g(x,u)$$

State estimation



$$\min_{x_0, w(t)} \int_{-T}^{0} |y - g(x,u)|_R^2 + |\dot{x} - f(x,u)|_Q^2 \, dt$$

$$\dot{x} = f(x,u) + w \quad \text{(process noise)}$$
$$y = g(x,u) + v \quad \text{(measurement noise)}$$

From optimal control to optimal *feedback* control

- Optimal control is developed in the 1960s: adjoint equations, Hamiltonian, maximum principle, etc. Treats nonlinear and constrained systems. (Bryson and Ho, 1975)
- Optimal *feedback* control for *linear, unconstrained* systems also developed in the 1960s: linear quadratic regulator (Kalman, 1960; Kwakernaak and Sivan, 1972)

$$u^0 = Kx$$

- Industrial systems are either constrained or nonlinear or both. Optimal *feedback* control for these systems seems to lead to *intractable* dynamic programming problems. The curse of dimensionality. (Bellman and Dreyfus, 1962)
- Optimal feedback control sees limited industrial application during this period.

So what unchained optimal control?

One technique for obtaining a feedback controller synthesis from knowledge of open-loop controllers is to measure the current control process state and then compute very rapidly for the open-loop control function. The first portion of this function is then used during a short time interval, after which a new measurement of the process state is made and a new open-loop control function is computed for this new measurement. The procedure is then repeated.

— Lee and Markus (1967)
Foundations of Optimal Control Theory

Our notion of very rapidly changed radically from 1960 to 1985.

Large industrial success story!

Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity
- 30-50 M$/year increased profit due to increased throughput (2008)

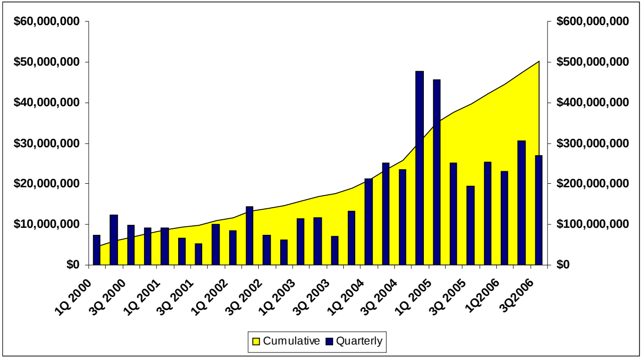
Praxair experience with MPC

- Praxair currently has more than 150 MPC installations
- 16 M$/year increased profit (2008)

Impact for 13 ethylene plants (Starks and Arrieta, 2007)

Hydrocarbons AC&O 17

We're Doing it For the Money



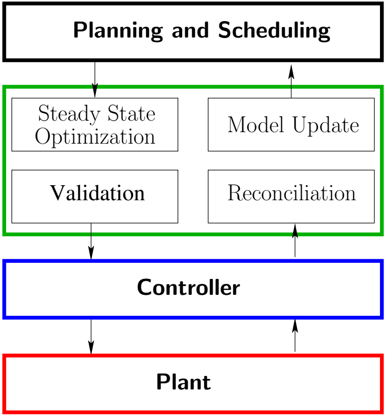
Advanced Control & Optimization

Are all the problems solved?

Some questions to consider

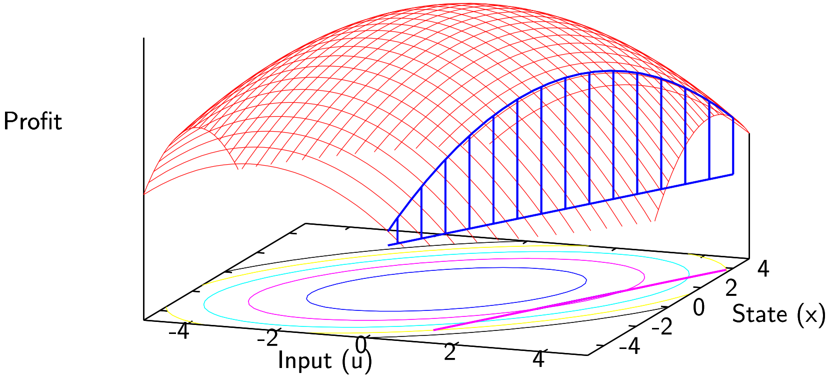
- Has the application base stopped growing?
- Is the theory complete?
- Do we have tools to solve *nonconvex optimization problems* online?
- Do we have tools to decompose *large-scale systems* into manageable problems?
- Do we have tools to *commission* and *maintain* the controllers?
- Do we have tools to optimize dynamic *economic* operation?

Optimizing economics: Current industrial practice



- Two layer structure
- Drawbacks
  - Inconsistent models
  - Re-identify linear model as setpoint changes
  - Time scale separation may not hold
  - Economics unavailable in dynamic layer

Optimizing economics: what's desirable?



Steady-state problem definition

- Stage cost:

$$\ell(x, u) = \text{plant profit function}$$

- Optimization: $\min_{x,u} \ell(x, u)$
  subject to: $x = f(x, u) \qquad x \in \mathbb{X} \quad u \in \mathbb{U}$
- Solution: (x_s, u_s)

MPC problem definition — dynamic case

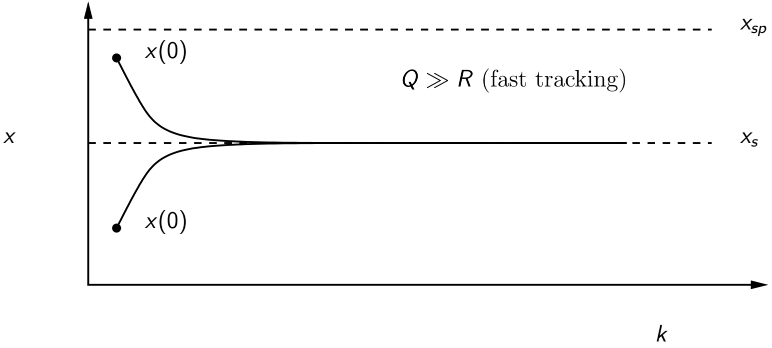
- Cost function: $$V = \sum_{j=0}^{N-1} \tilde{\ell}(x(j), u(j))$$
- Optimization: $$\min_{\mathbf{u}} V(\mathbf{u}, x(0))$$
  subject to:
  $$\mathbf{u} = \{u(0), u(1), \dots u(N-1)\}$$
  $$x^+ = f(x, u) \qquad \mathbf{x} \in \mathbb{X}^N \qquad \mathbf{u} \in \mathbb{U}^N$$
- Stage cost:

  std–MPC: $\tilde{\ell}(x, u) = |x - x_s|_Q^2 + |u - u_s|_R^2$ –or–

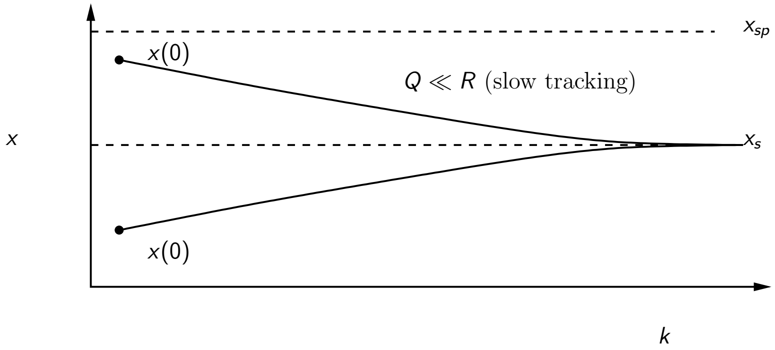
  eco–MPC: $\tilde{\ell}(x, u) = \ell(x, u)$

- Control law: $u^0(x) = \mathbf{u}^0(0; x)$

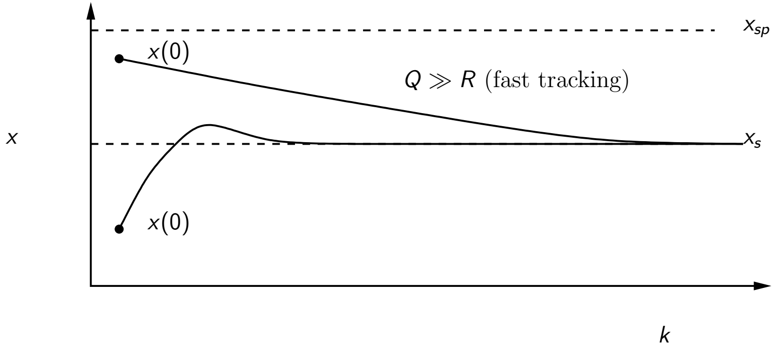
What closed-loop behavior is desirable? Fast tracking



What closed-loop behavior is desirable? Slow tracking



What closed-loop behavior is desirable? Asymmetric tracking



Unreachable case — challenges for analyzing closed-loop behavior

- Sequence of optimal costs is not monotone decreasing
- Infinite horizon cost is unbounded for all input sequences
- Optimal cost is not a Lyapunov function for the closed-loop system
- Standard nominal MPC stability arguments do not apply
- Simulations indicate the closed loop *is* stable
- How can we be sure?

Recent results for economic MPC

- The economic MPC for linear dynamics, $f(x, u) = Ax + Bu$, and convex cost is asymptotically stabilizing (Rawlings, Bonné, Jørgensen, Venkat, and Jørgensen, 2008)
- Proof based on convexity of stage cost
- No Lyapunov function was found

A brief history of this paper

- John Jørgensen visits Madison. January–August 2000
- Dennis Bonné visits Madison. February–August 2002
- Paper is submitted to IEEE TAC. July 2007
- Paper is submitted again. November 2007
- First reviews come back. February 2008
- Revised paper submitted. February 2008
- Paper is accepted. April 2008
- Paper appears. October 2008
- 8 1/2 years since John first arrived in Madison

Some funny emails during this long journey

```
From: "James B. Rawlings" <jbraw@bahaha.che.wisc.edu>
To: John Bagterp <bagterp@bevo.che.wisc.edu>
Date: Wed, 2 Aug 2000 07:10:34 -0500 (CDT)

p.s. If Sten doesn't get excited about the approach
pretty soon, you should check his food;
maybe he's on some medication.
```

Sten speaks

```
On  3-Jul-2007, Sten Bay Jorgensen <SBJ@kt.dtu.dk> wrote:

I actually think this will be a very worthwhile
contribution  which indeed bridges
theory and application.
```

Rotated cost and Lyapunov function

- Stability results extended to nonlinear systems satisfying strong duality. There exists constant $\lambda \in \mathbb{R}^n$ such that

$$\min_{(x,u)\in\mathbb{Z}} \ell(x,u) + \lambda'(x - f(x,u)) \geq \ell(x_s, u_s)$$

- The Lyapunov function is based on rotated stage cost

$$L(x,u) = \ell(x,u) - \ell(x_s,u_s) + \lambda'(x - f(x,u))$$

- Diehl, Amrit, and Rawlings (2011)

Generalization to dissipative systems

- Stability results generalized to dissipative systems. There exists function $\lambda : \mathbb{X} \to \mathbb{R}$ and supply rate $s : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ such that

$$\lambda(f(x,u)) - \lambda(x) \leq s(x,u) \qquad (x,u) \in \mathbb{Z}$$

$$\min_{(x,u)\in\mathbb{Z}} \ell(x,u) + \lambda(x) - \lambda(f(x,u)) \geq \ell(x_s, u_s)$$

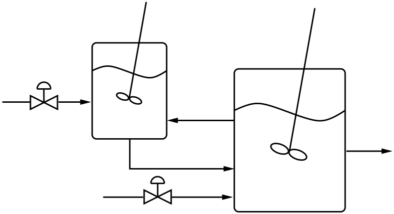
- Rotated stage cost

$$L(x,u) = \ell(x,u) - \ell(x_s,u_s) + \lambda(x) - \lambda(f(x,u))$$

- MPC also shown to outperform optimal periodic control
- Angeli, Amrit, and Rawlings (2011)

Nonlinear chemical reactor example

$$P_0 + B \longrightarrow P_1$$
$$P_1 + B \longrightarrow P_2$$

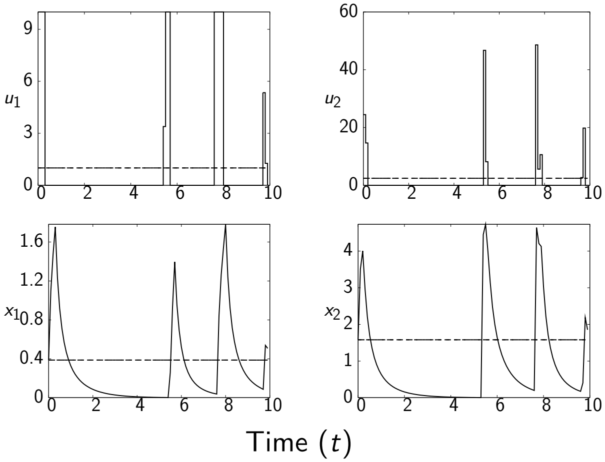


Species mass balances:

$$\dot{x}_1 = u_1 - x_1 - \sigma_1 x_1 x_2$$
$$\dot{x}_2 = u_2 - x_2 - \sigma_1 x_1 x_2 - \sigma_2 x_2 x_3$$
$$\dot{x}_3 = -x_3 + \sigma_1 x_1 x_2 - \sigma_2 x_2 x_3$$
$$\dot{x}_4 = -x_4 + \sigma_2 x_2 x_3$$

x_1, x_2, x_3, x_4: concentrations of P_0, B, P_1, P_2
u_1, u_2: inflow rates of P_0 and B

Nonlinear chemical reactor example



Enforcing convergence

- Optimizing economic performance may not give stability
- Tradeoff between stability and performance
- Convergence may be enforced by modifying the dynamic problem
  - Convex regularization terms in the objective

$$\tilde{\ell}(x, u) = \ell(x, u) + |x - x_s|_Q^2 + |u - u_s|_R^2$$

  - Zero variance constraint

$$\mathrm{Av}[|x - x_s|^2] \in \{0\}$$

Enforcing convergence: Chemical reactor example

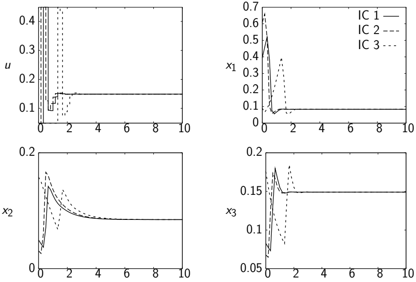
CSTR with parallel reactions

$$R \rightarrow P_1$$
$$R \rightarrow P_2$$

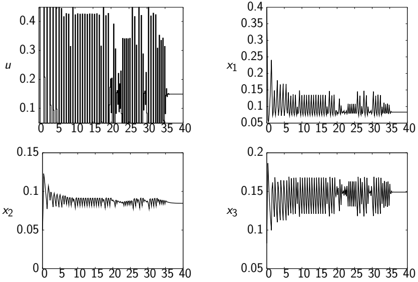
$$\dot{x}_1 = 1 - 10^4 x_1^2 e^{-1/x_3} - 400 x_1 e^{-0.55/x_3} - x_1$$
$$\dot{x}_2 = 10^4 x_1^2 e^{-1/x_3} - x_2$$
$$\dot{x}_3 = u - x_3$$

x_1, x_2: Concentrations of R, P_1
x_3: Temperature in the reactor.
u: Heat flux through the reactor wall

- Convex terms in objective



- Convergence constraint



Status of Economic MPC

- Opportunities
  - Performance advantage
  - Consistent model for optimizing performance
  - Consistent statement of process objectives
  - Optimization software well developed
  - Linear dynamics and convex objectives well supported
  - Leverage from industrial implementation of linear MPC
  - This is a big opportunity!

Status of Economic MPC

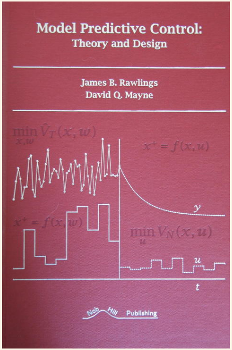
- Challenges
  - Understanding the interplay between nonlinearity of model and convexity of objective
  - Managing the complexity of an optimal *economic* solution
  - Developing theory, algorithms, tuning procedures, and realistic case studies that support industrial implementation
  - This is a big challenge!

MPC Research Monograph

Writing a book is an adventure. To begin with it is a toy, then amusement, then it becomes a mistress, then it becomes a master, and then it becomes a tyrant, and the last phase is that just as you are about to become reconciled to your servitude, you kill the monster and strew him about to the public.

– Winston Churchill

MPC Research Monograph



- Published in 2009
- 576 page text
- 3 appendices on web (133 pages)
- 214 exercises
- 335 page solution manual
- total: 1044 pages
- www.nobhillpublishing.com

Some (of many!) connections between UW Madison and DTU

- Collaboration of John Villadsen (DTU) and Warren Stewart (UW) on orthogonal collocation (Villadsen and Stewart, 1967)
- Book by John Villadsen and Michael Michelsen (Villadsen and Michelsen, 1978)
- Ole Hassager (DTU); PhD (1973) in polymers with Bob Bird (UW)
- Juan de Pablo (UW); 2011 Ørsted Lectureship Award from DTU
- Manos Mavrikakis (UW) taught Catalytic and Advanced Reaction Engineering course at DTU in 2006
- Sten Bay Jørgensen and the systems area
- John Jørgensen and Dennis Bonné visited UW during their PhDs
- Emil Sokoler visited UW during his PhD and worked with me and Steve Wright, computer science. Third generation of interactions!

Further reading I

D. Angeli, R. Amrit, and J. B. Rawlings. On average performance and stability of economic model predictive control. *IEEE Trans. Auto. Cont.*, 2011. Accepted for publication.

R. E. Bellman and S. E. Dreyfus. *Applied Dynamic Programming*. Princeton University Press, Princeton, New Jersey, 1962.

A. E. Bryson and Y. Ho. *Applied Optimal Control*. Hemisphere Publishing, New York, 1975.

M. Diehl, R. Amrit, and J. B. Rawlings. A Lyapunov function for economic optimizing model predictive control. *IEEE Trans. Auto. Cont.*, 56(3):703–707, 2011.

R. E. Kalman. Contributions to the theory of optimal control. *Bull. Soc. Math. Mex.*, 5:102–119, 1960.

H. Kwakernaak and R. Sivan. *Linear Optimal Control Systems*. John Wiley and Sons, New York, 1972. ISBN 0-471-51110-2.

Further reading II

J. B. Rawlings, D. Bonné, J. B. Jørgensen, A. N. Venkat, and S. B. Jørgensen. Unreachable setpoints in model predictive control. *IEEE Trans. Auto. Cont.*, 53 (9):2209–2215, October 2008.

D. M. Starks and E. Arrieta. Maintaining AC&O applications, sustaining the gain. In *Proceedings of National AIChE Spring Meeting*, Houston, Texas, April 2007.

J. Villadsen and M. L. Michelsen. *Solution of Differential Equation Models by Polynomial Approximation*. Prentice-Hall, Englewood Cliffs New Jersey, 1978.

J. V. Villadsen and W. E. Stewart. Solution of boundary-value problems by orthogonal collocation. *Chem. Eng. Sci.*, 22:1483–1501, 1967.