Controlling Large-Scale Systems with Distributed Model Predictive Control

James B. Rawlings

Department of Chemical and Biological Engineering

THE UNIVERSITY of WISCONSIN MADISON

November 8, 2010
Annual AIChE Meeting
Salt Lake City, UT

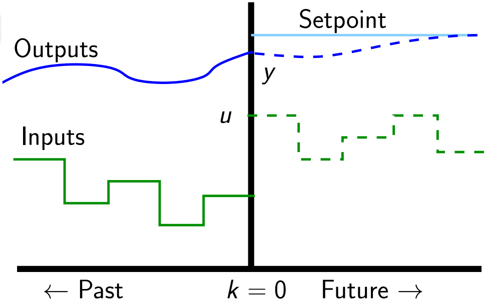
Outline

1. Overview of Distributed Model Predictive Control
   - Control of large-scale systems
2. Cooperative Control
   - Stability theory for cooperative MPC
3. Conclusions and Future Outlook
4. Some Comments on Tom Edgar

Model predictive control

What are the goals of MPC?

- Choose inputs which bring outputs to their setpoints
- Minimize objective function over N future steps



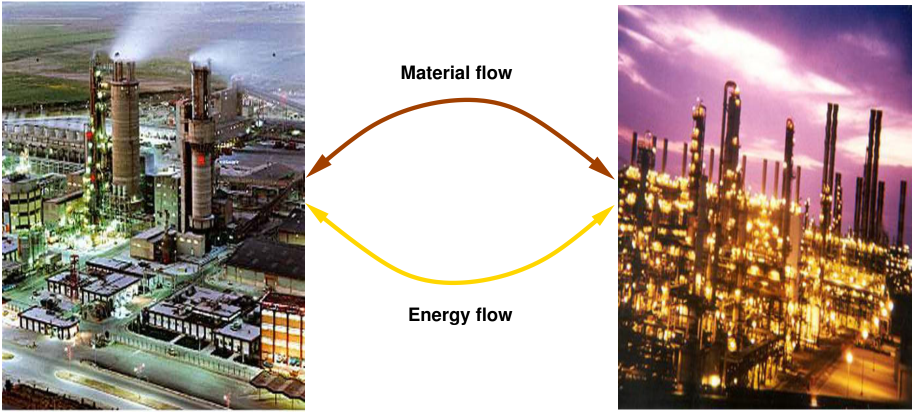
$$\min_{\mathbf{u}} V(x, \mathbf{u}) = \sum_{k=0}^{N-1} \ell(x(k), u(k)) + V_f(x(N))$$

subject to

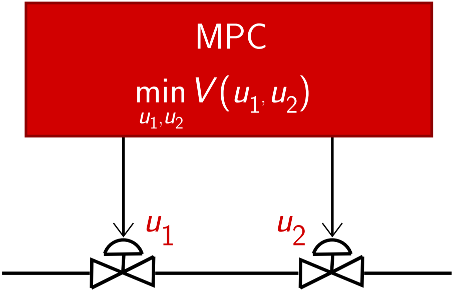
$$x^+ = Ax + Bu$$

$$y = Cx$$

Chemical plant integration

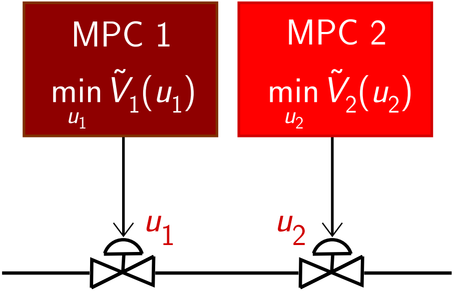


Ideal plantwide MPC



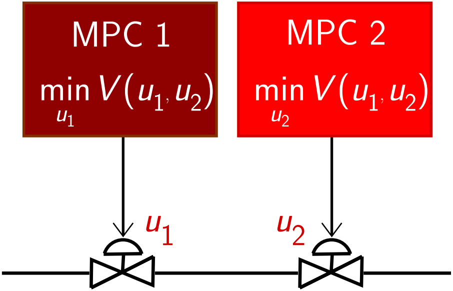
- Ideal controller
  - perfect model
  - never goes offline
  - optimizes infinitely fast
  - samples infinitely fast

Practical plantwide MPC



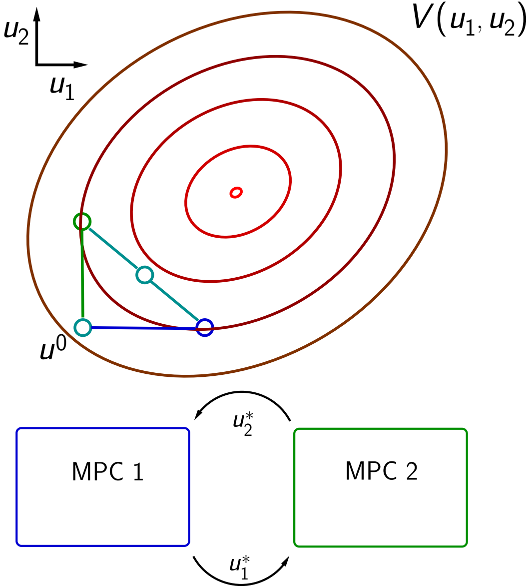
- Realistic controller
  - approximate model
  - MPCs fail or require maintenance
  - finite optimization time
  - multiple sampling rates
- Goal: make realistic controller close to ideal controller

Plantwide distributed MPC

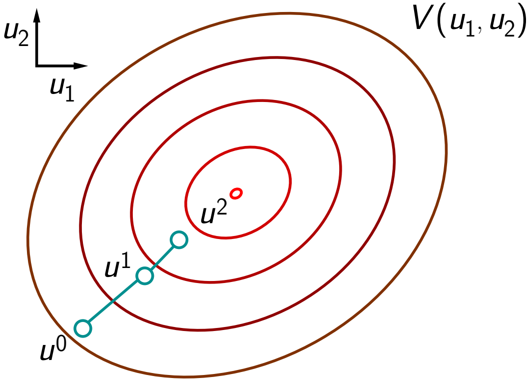


- Decentralized control
  - no communication
  - not stable for strongly interacting subsystems
- Noncooperative control
  - use full modeling information
  - not stable for strongly interacting subsystems
- Cooperative control
  - use same objective in each controller
  - stability independent of interaction strength

Cooperative model predictive control



Plantwide suboptimal MPC



- Early termination of optimization gives suboptimal plantwide feedback
- Use suboptimal MPC theory to prove stability

Plantwide suboptimal MPC

Consider closed-loop system augmented with input trajectory

$$\begin{pmatrix} x^+ \\ \mathbf{u}^+ \end{pmatrix} = \begin{pmatrix} Ax + Bu \\ g(x, \mathbf{u}) \end{pmatrix}$$

- Function $g(\cdot)$ returns suboptimal choice
- Stability of augmented system is established by Lyapunov function

$$a\,|(x, \mathbf{u})|^2 \leq V(x, \mathbf{u}) \leq b\,|(x, \mathbf{u})|^2$$
$$V(x^+, \mathbf{u}^+) - V(x, \mathbf{u}) \leq -c\,|(x, u)|^2$$

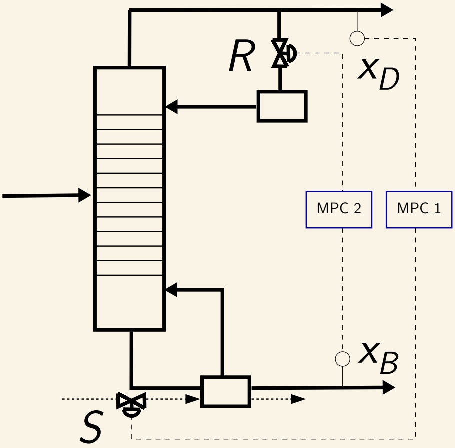
- Adding constraint establishes closed-loop stability of the origin for all $\mathbf{u}$[1]

$$|\mathbf{u}| \leq d\,|x| \quad x \in \mathbb{B}_r, r > 0$$

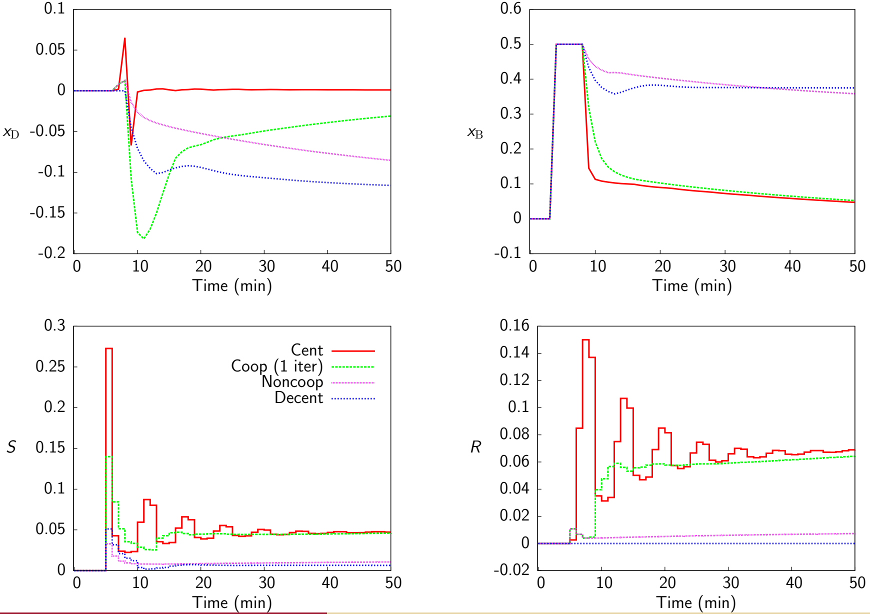
- Cooperative optimization satisfies these properties for plantwide objective function $V(x, \mathbf{u})$

[1](Rawlings and Mayne, 2009, pp.418-420)

LV control of distillation column



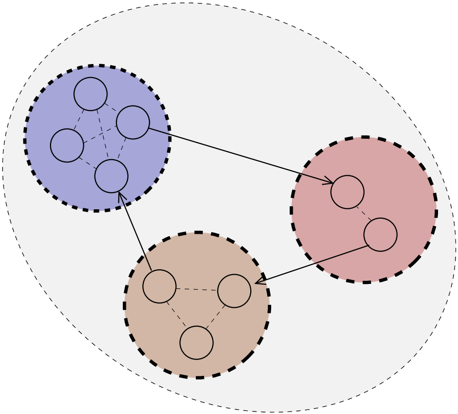
LV control of distillation column



Performance comparison

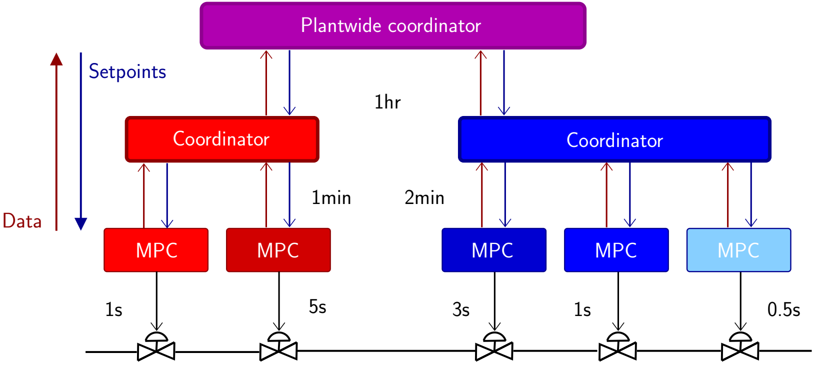
	Cost	Performance loss (%)
Centralized MPC	75.8	0
Cooperative MPC (10 iterates)	76.1	0.388
Cooperative MPC (1 iterate)	87.5	15.4
Noncooperative MPC	382	404
Decentralized MPC	364	380

Plantwide topology



- Plant subsystems can often be grouped spatially or dynamically
- Neighborhoods of subsystems naturally arise from topology

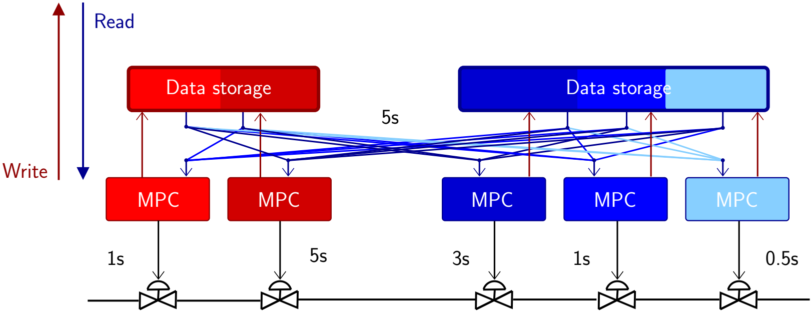
Traditional hierarchical MPC[2]



- Multiple dynamical time scales in plant
- Data and setpoints are exchanged on chosen scale
- Optimization performed at each layer

[2] Mesarović et al. (1970); Scattolini (2009)

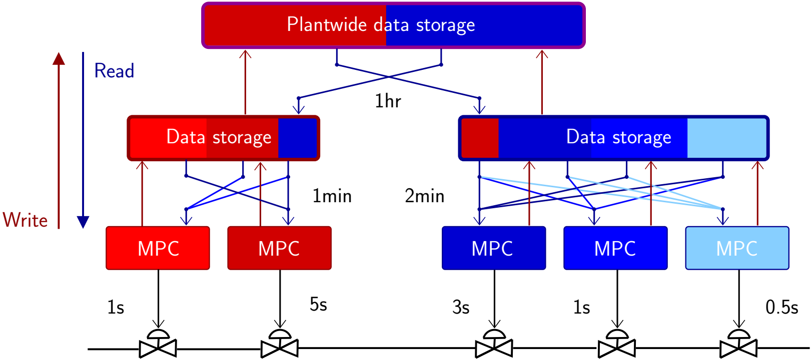
Cooperative MPC data exchange[3]



- All data exchanged plantwide
- Data exchange at each controller execution

[3]Venkat (2006); Stewart et al. (2010b)

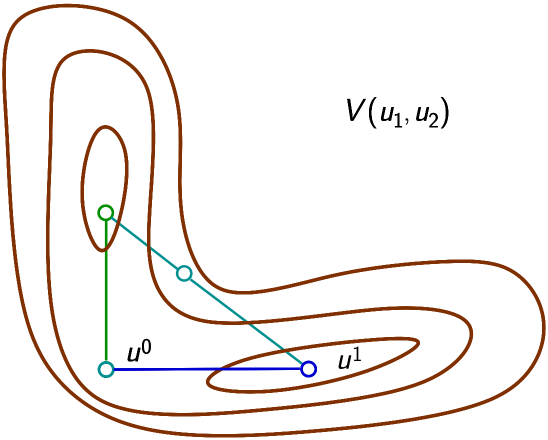
Cooperative hierarchical MPC[4]



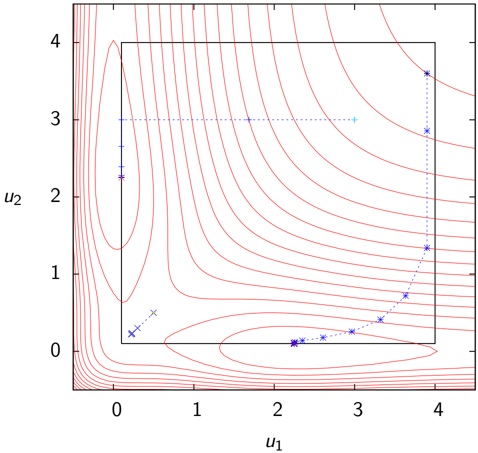
- Optimization at MPC layer only
- Only subset of data exchanged plantwide
- Data exchanged at chosen time scale

[4] Stewart et al. (2010a)

The challenge of nonlinear models



Distributed gradient projection - example



$$V(u_1, u_2) = e^{-2u_1} - 2e^{-u_1} + e^{-2u_2} - 2e^{-u_2}$$
$$+1.1\exp(-0.4((u_1+0.2)^2 + (u_2+0.2)^2))$$

Conclusions

Cooperative MPC theory maturing (Stewart et al., 2010b; Maestre et al., 2010)

- Satisfies hard input constraints
- Provides nominal stability for plants with even strongly interacting subsystems
- Retains closed-loop stability for early iteration termination
- Converges to Pareto optimal control in the limit of iteration
- Remains stable under perturbation from stable state estimator
- Avoids coordination layer

Future Outlook

Extensions required for practical implementation

- Can we treat nonlinear plant models? Qualified yes.
- Can we avoid coupled constraints? Qualified yes.
- Can we reduce the assumed complete communication? Yes.
- Can we accommodate time-scale separation? Yes.
- Can we nest layers within layers? Yes.

Further reading I

J. M. Maestre, D. Muñoz de la Peña, and E. F. Camacho. Distributed model predictive control based on a cooperative game. *Optimal Cont. Appl. Meth.*, In press, 2010.

M. Mesarović, D. Macko, and Y. Takahara. *Theory of hierarchical, multilevel systems*. Academic Press, New York, 1970.

J. B. Rawlings and D. Q. Mayne. *Model Predictive Control: Theory and Design*. Nob Hill Publishing, Madison, WI, 2009. 576 pages, ISBN 978-0-9759377-0-9.

R. Scattolini. Architectures for distributed and hierarchical model predictive control - a review. *J. Proc. Cont.*, 19(5):723–731, May 2009. ISSN 0959-1524.

B. T. Stewart, J. B. Rawlings, and S. J. Wright. Hierarchical cooperative distributed model predictive control. In *Proceedings of the American Control Conference*, Baltimore, Maryland, June 2010a.

B. T. Stewart, A. N. Venkat, J. B. Rawlings, S. J. Wright, and G. Pannocchia. Cooperative distributed model predictive control. *Sys. Cont. Let.*, 59:460–469, 2010b.

A. N. Venkat. *Distributed Model Predictive Control: Theory and Applications*. PhD thesis, University of Wisconsin–Madison, October 2006. URL `http://jbrwww.che.wisc.edu/theses/venkat.pdf`.