Optimizing Process Economic Performance with Model Predictive Control

James B. Rawlings

Department of Chemical and Biological Engineering

THE UNIVERSITY of WISCONSIN MADISON

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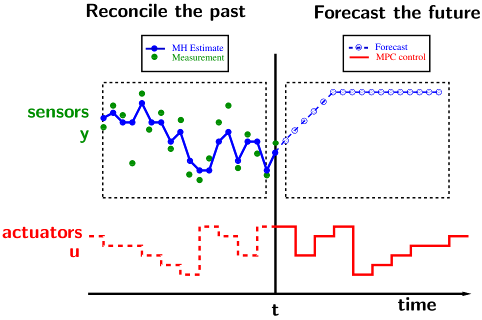
University of Stuttgart

Outline

1. Optimal control, optimal feedback control, and model predictive control (MPC)
2. Industrial impact of these ideas
3. Using MPC to optimize plant economics

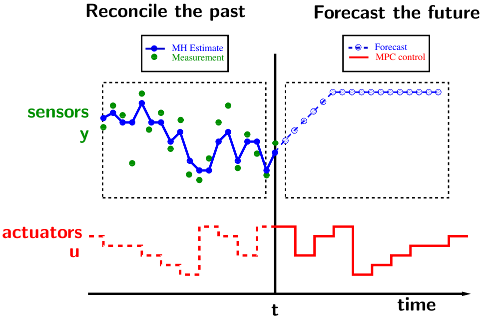
Predictive control



$$\min_{u(t)} \int_0^T |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2 \, dt$$

$$\dot{x} = f(x,u)$$
$$x(0) = x_0 \quad \text{(given)}$$
$$y = g(x,u)$$

State estimation



$$\min_{x_0, w(t)} \int_{-T}^{0} |y - g(x,u)|_R^2 + |\dot{x} - f(x,u)|_Q^2 \, dt$$

$$\begin{aligned} \dot{x} &= f(x,u) + w \quad \text{(process noise)} \\ y &= g(x,u) + v \quad \text{(measurement noise)} \end{aligned}$$

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- Optimal feedback control sees limited industrial application during this period.

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One technique for obtaining a feedback controller synthesis from knowledge of open-loop controllers is to measure the current control process state and then compute very rapidly for the open-loop control function.

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— *Lee and Markus (1967)*
Foundations of Optimal Control Theory

Our notion of very rapidly changed radically from 1960 to 1985.

Large industrial success story!

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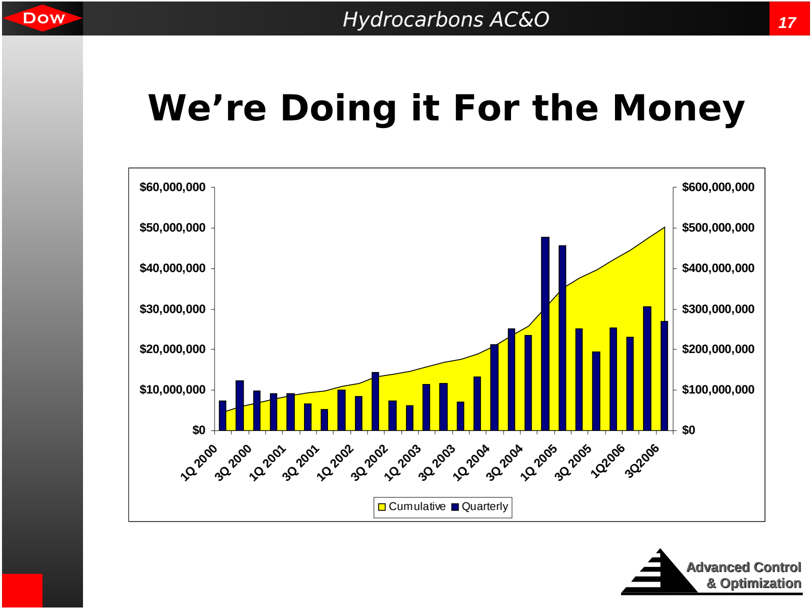
Impact for 13 ethylene plants (Starks and Arrieta, 2007)



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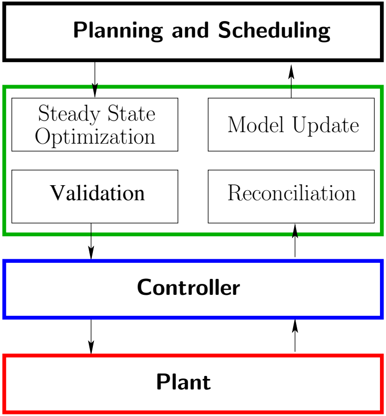
- Do we have tools to optimize dynamic *economic* operation?

Optimizing economics: Current industrial practice



- Two layer structure
- Drawbacks

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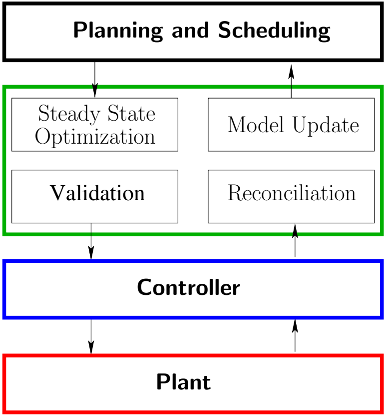
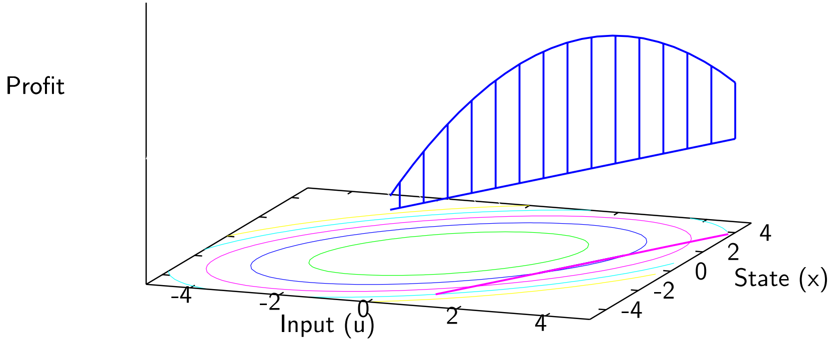
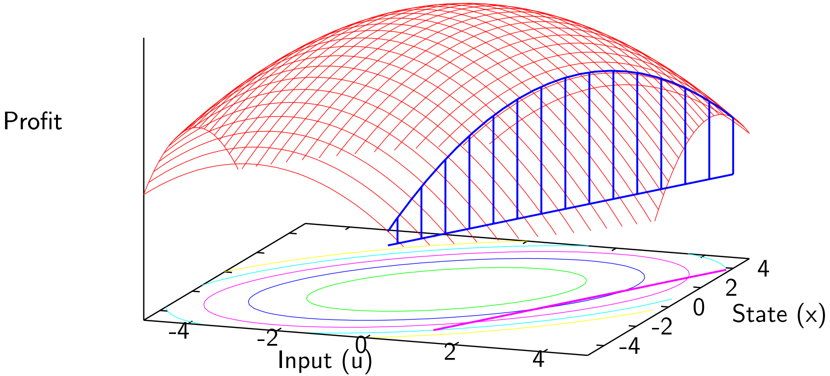


- Two layer structure
- Drawbacks
  - Inconsistent models
  - Re-identify linear model as setpoint changes
  - Time scale separation may not hold
  - Economics unavailable in dynamic layer

Optimizing economics: what's desirable?



Optimizing economics: what's desirable?



Steady-state problem definition

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- Solution: (x_s, u_s)

MPC problem definition — dynamic case

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  subject to:

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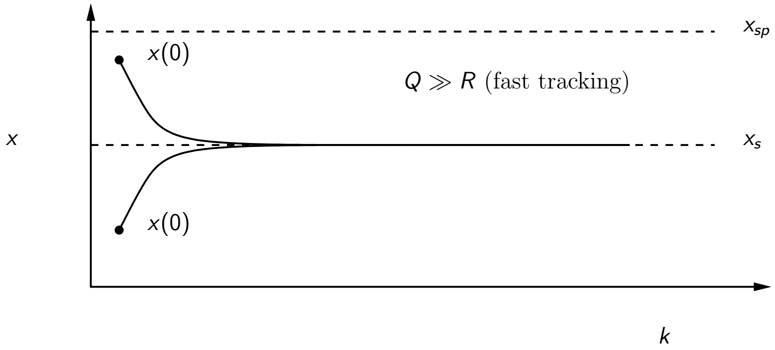
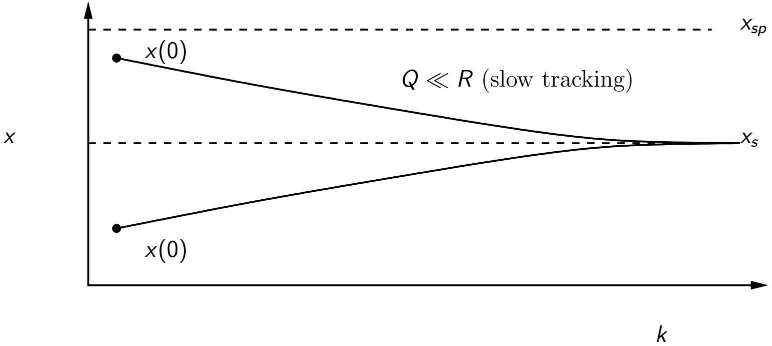
- Control law: $u^0(x) = \mathbf{u}^0(0; x)$

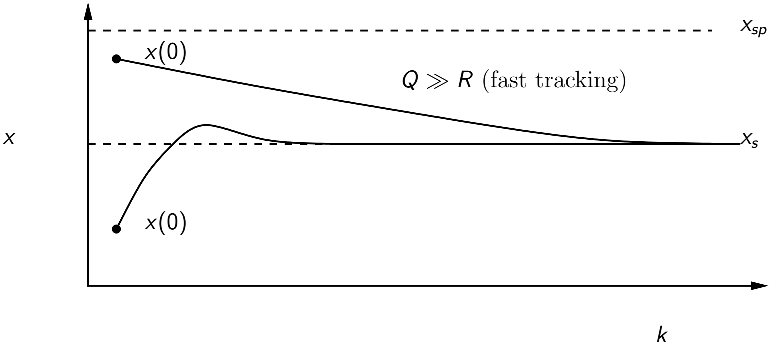
What closed-loop behavior is desirable? Fast tracking



What closed-loop behavior is desirable? Slow tracking



What closed-loop behavior is desirable? Asymmetric tracking



Unreachable case — challenges for analyzing closed-loop behavior

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- Simulations indicate the closed loop *is* stable
- How can we be sure?

Recent results for economic MPC

- The economic MPC for linear dynamics, $f(x,u) = Ax + Bu$, and convex cost is asymptotically stabilizing (Rawlings, Bonné, Jørgensen, Venkat, and Jørgensen, 2008)

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- The economic MPC for linear dynamics, $f(x, u) = Ax + Bu$, and convex cost is asymptotically stabilizing (Rawlings, Bonné, Jørgensen, Venkat, and Jørgensen, 2008)
- Proof based on convexity of stage cost
- No Lyapunov function was found

Rotated cost and Lyapunov function

- Stability results extended to nonlinear systems satisfying strong duality. There exists constant $\lambda \in \mathbb{R}^n$ such that

$$\min_{(x,u)\in\mathbb{Z}} \ell(x,u) + \lambda'(x - f(x,u)) \geq \ell(x_s, u_s)$$

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- The Lyapunov function is based on rotated stage cost

$$L(x,u) = \ell(x,u) - \ell(x_s,u_s) + \lambda'(x - f(x,u))$$

- Diehl, Amrit, and Rawlings (2011)

Generalization to dissipative systems

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$$\lambda(f(x,u)) - \lambda(x) \leq s(x,u) \qquad (x,u) \in \mathbb{Z}$$

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$$\min_{(x,u)\in\mathbb{Z}} \ell(x,u) + \lambda(x) - \lambda(f(x,u)) \geq \ell(x_s, u_s)$$

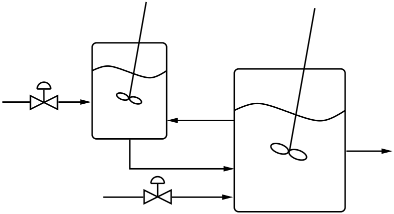
- Rotated stage cost

$$L(x,u) = \ell(x,u) - \ell(x_s,u_s) + \lambda(x) - \lambda(f(x,u))$$

- MPC also shown to outperform optimal steady performance
- Angeli, Amrit, and Rawlings (2011)

Nonlinear chemical reactor example

$$P_0 + B \longrightarrow P_1$$
$$P_1 + B \longrightarrow P_2$$



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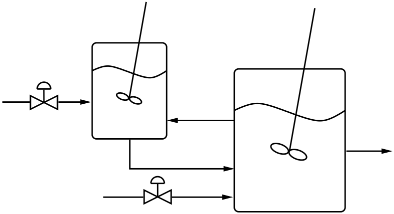
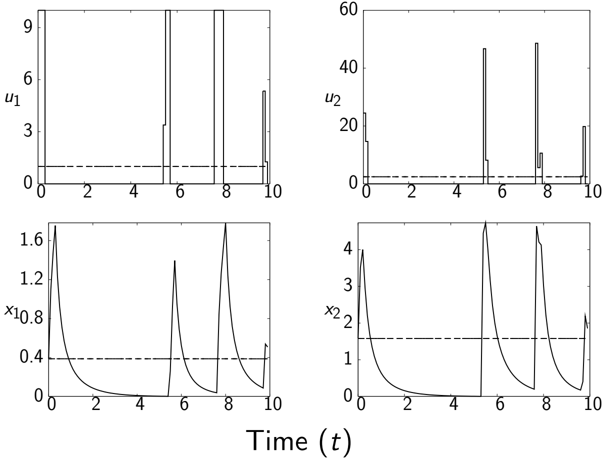


Species mass balances:

$$\dot{x}_1 = u_1 - x_1 - \sigma_1 x_1 x_2$$
$$\dot{x}_2 = u_2 - x_2 - \sigma_1 x_1 x_2 - \sigma_2 x_2 x_3$$
$$\dot{x}_3 = -x_3 + \sigma_1 x_1 x_2 - \sigma_2 x_2 x_3$$
$$\dot{x}_4 = -x_4 + \sigma_2 x_2 x_3$$

x_1, x_2, x_3, x_4: concentrations of P_0, B, P_1, P_2
u_1, u_2: inflow rates of P_0 and B

Nonlinear chemical reactor example: Unstable system



- System initialized at the best steady state
- Steady solution not optimal: Unstable system !

Average performance

- Asymptotic average of any function of states and inputs $(y = h(x, u))$:

$$\text{Av}[y] := \lim_{T\to\infty} \frac{\sum\limits_{k=0}^{T} h(x(k), u(k))}{T+1}$$

- Asymptotic average performance along the closed loop

$$\text{Av}\left[\ell(x, \mathbf{u})\right] := \lim_{T\to\infty} \frac{\sum_{k=0}^{T} \ell(x(k), u(k))}{T+1}, \quad x^{+} = f(x, u)$$

Nonsteady operation: Average constraints

- Auxiliary output variable $y = h(x, u)$
- Closed and convex set $\mathbb{Y}$, such that $h(x_s, u_s) \in \mathbb{Y}$

- Receding horizon control strategy such that

$$(x(k), u(k)) \in \mathbb{Z} \quad k \in \mathbb{I}_{\geq 0}$$
$$\mathrm{Av}[y] \subseteq \mathbb{Y}$$

Average constraints: Algorithm

1. At each time t solve the following optimization problem

$$\min_{\mathbf{v}} \sum_{k=0}^{N-1} \ell(z(k), v(k))$$

$$z^+ = f(z, v)$$

$$(z(k), v(k)) \in \mathbb{Z}, \quad k \in \mathbb{I}_{0:N-1}$$

$$z(N) = x_s, \quad z(0) = x$$

$$\sum_{k=0}^{N-1} h(z(k), v(k)) \in \mathbb{Y}_t$$

2. The time-varying output constraint set defined recursively

$$\mathbb{Y}_{i+1} = \mathbb{Y}_i \oplus \mathbb{Y} \ominus h(x(i), u(i)) \qquad \text{for } i \in \mathbb{I}_{\geq 0}$$

$$\mathbb{Y}_0 = N\mathbb{Y} + \mathbb{Y}_{00}, \quad \mathbb{Y}_{00} \subset \mathbb{R}^{N_y}$$

3. Inject the first move of the optimal solution into the system and goto step 1

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$$P_0 + B \longrightarrow P_1$$
$$P_1 + B \longrightarrow P_2$$

Constraint: $\text{Av}[u_1] \in [0, 1]$

Economic objective: Maximize $P_1(x_3)$

Species mass balances:

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- Unstable solution
- $\mathrm{Av}[u_1] = 1$
- Average performance beats best steady state
  $\mathrm{Av}[x_3] = 0.44 \geq x_{3s} = 0.37$

Species mass balances:

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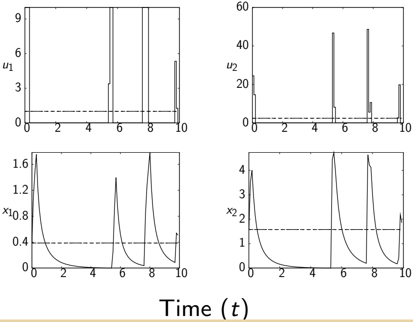

Nonsteady operation: Periodic operation

- Economics optimizing MPC may not be stable
  - Nonsteady operation outperforms best steady state solution

$$\text{Av}[\ell(x, \mathbf{u}^0)] \leq \ell(x_s, u_s), \quad \forall x \in \mathcal{X}_N$$

- Periodic operation
  - Known period of operation: Q
  - Periodic constraint: $x(0) = x(Q)$
  - Optimal periodic solution $x^\star(k)$, $k \in \mathbb{I}_{0:Q-1}$

$$\min_{x(0),\mathbf{u}} V_Q(x(0), \mathbf{u}) = \sum_{k=0}^{Q-1} \ell(x(k), u(k))$$

$$\text{subject to} \begin{cases} x^+ = f(x, u) \\ (x(k), u(k)) \in \mathbb{Z}, \quad k \in \mathbb{I}_{0:Q-1} \\ x(Q) = x(0) \end{cases}$$

Nonsteady operation: Periodic MPC

- Open loop dynamic regulation problem at time t

$$\min_{\mathbf{v}} \sum_{k=0}^{N-1} \ell(z(k), v(k))$$

$$z^+ = f(z, v), \quad z(0) = x$$

$$(z(k), v(k)) \in \mathbb{Z}, \quad k \in \mathbb{I}_{0:N-1}$$

$$z(N) = x^\star(t \bmod Q)$$

Closed loop control law: $u_P^0(x) = u^0(0; x)$

- Periodic MPC outperforms optimal periodic solution

$$\mathrm{Av}[\ell(x, \mathbf{u}_P^0)] \leq \frac{\sum_{k=0}^{Q-1} \ell(x^\star(k), u^\star(k))}{Q}$$

Chemical reactor example: Parallel reactions

CSTR with parallel reactions

$$R \to P_1$$
$$R \to P_2$$

Economic objective: Maximize P_1 (x_2)

$$\dot{x}_1 = 1 - 10^4 x_1^2 e^{-1/x_3} - 400 x_1 e^{-0.55/x_3} - x_1$$
$$\dot{x}_2 = 10^4 x_1^2 e^{-1/x_3} - x_2$$
$$\dot{x}_3 = u - x_3$$

x_1, x_2: Concentrations of R, P_1
x_3: Temperature in the reactor.
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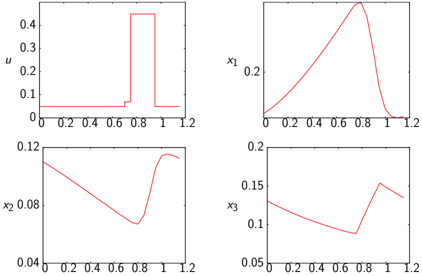
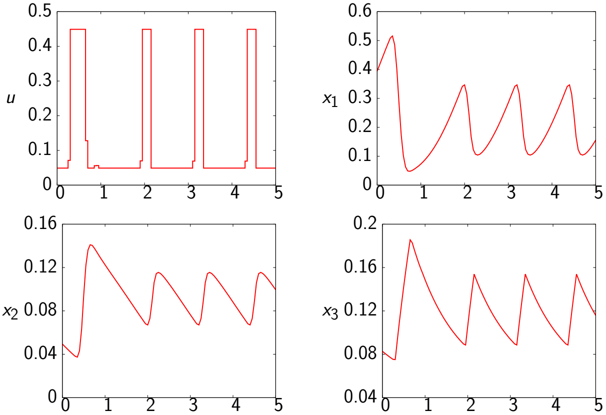
- Optimal steady state $x_{2s} = 0.084$
- Optimal periodic solution: $\text{Av}[x_2^\star(k)] = 0.092$





- System under closed loop settles to the recurring periodic solution
- Asymptotic average performance $\text{Av}[x_2] = 0.093$
- Average performance beats average periodic solution

Enforcing convergence

- Optimizing economic performance may not give stability
- Tradeoff between stability and performance
- Convergence may be enforced by modifying the dynamic problem
  - Convex regularization terms in the objective

$$\tilde{\ell}(x, u) = \ell(x, u) + |\Delta u|_S^2 + |x - x_s|_Q^2 + |u - u_s|_R^2$$

  - Zero variance constraint

$$\mathrm{Av}[|x - x_s|^2] \in \{0\}$$

Chemical reactor example: Parallel reactions

CSTR with parallel reactions

$$R \to P_1$$
$$R \to P_2$$

Economic objective: Maximize P_1 (x_2)

$$\dot{x}_1 = 1 - 10^4 x_1^2 e^{-1/x_3} - 400 x_1 e^{-0.55/x_3} - x_1$$
$$\dot{x}_2 = 10^4 x_1^2 e^{-1/x_3} - x_2$$
$$\dot{x}_3 = u - x_3$$

x_1, x_2: Concentrations of R, P_1

x_3: Temperature in the reactor.

u: Heat flux through the reactor wall

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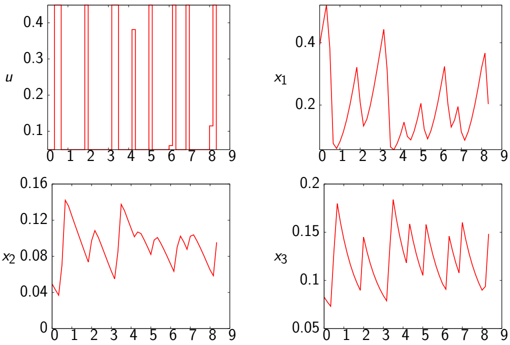

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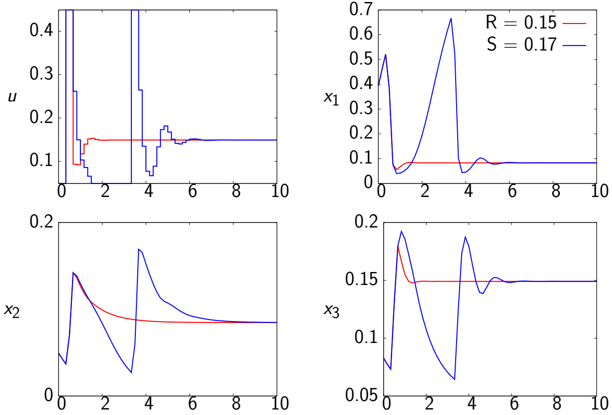

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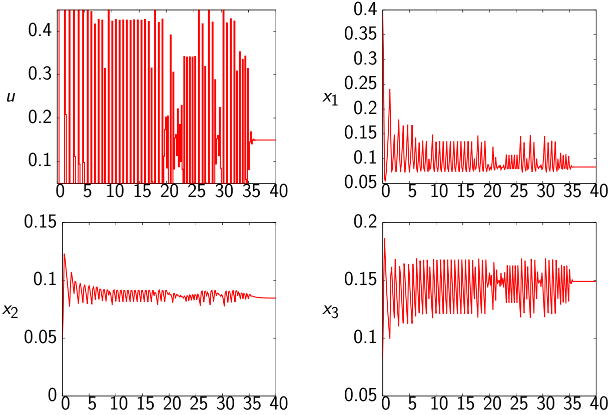

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  - Performance advantage

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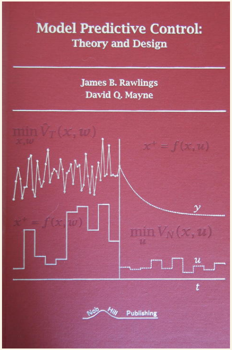

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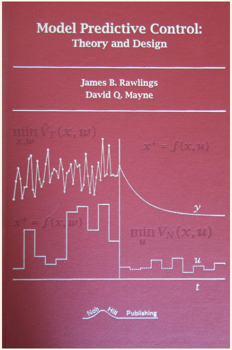

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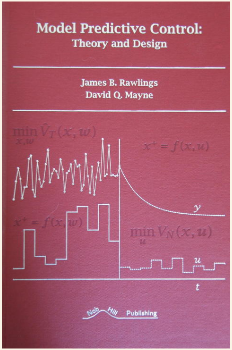

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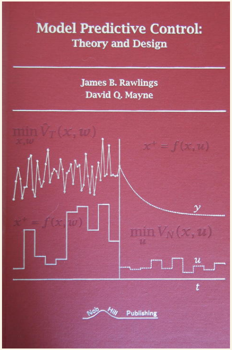

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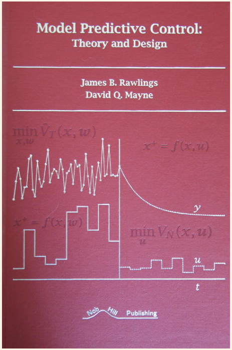

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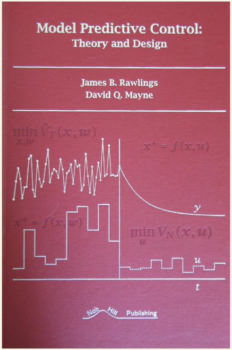

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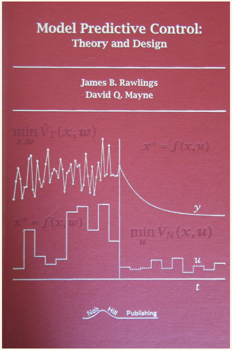

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