# Optimal dynamic operation of chemical processes: current opportunities

James B. Rawlings

Department of Chemical and Biological Engineering

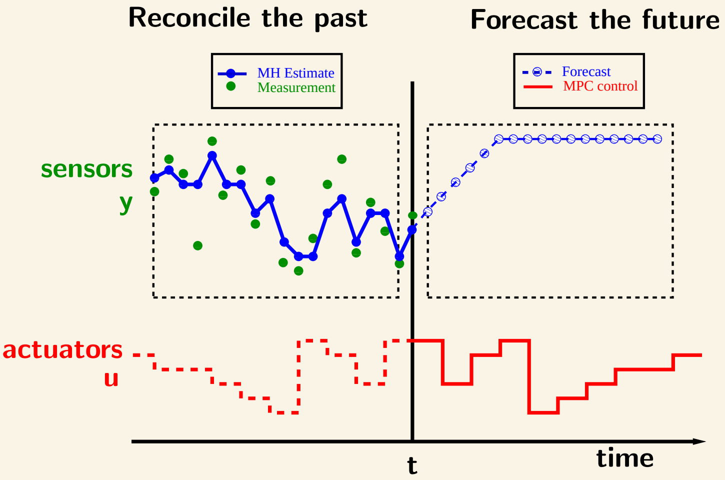
THE UNIVERSITY of WISCONSIN MADISON

August 6, 2009
Westhollow Research Center
Shell

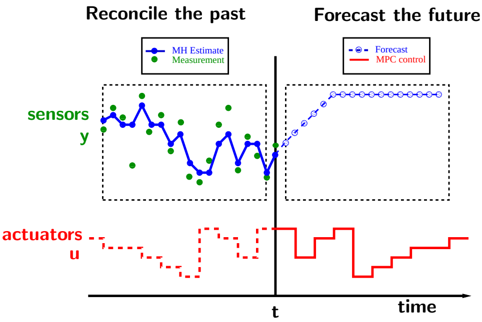
# Outline

1. Introductory overview to Model Predictive Control (MPC)
2. Industrial impact of these ideas
3. Recent developments
   - Distributed MPC
   - Large-scale systems and partial enumeration
   - Tools for controller commissioning
   - Optimizing economics
   - Continuous time MPC
4. Conclusions and future outlook
5. Further Reading

# The model predictive control framework



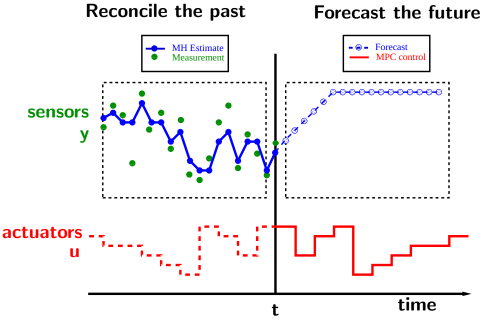
# Predictive control




$$\min_{u(t)} \int_0^T |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2 \, dt$$

$$\dot{x} = f(x,u)$$
$$x(0) = x_0 \quad \text{(given)}$$
$$y = g(x,u)$$

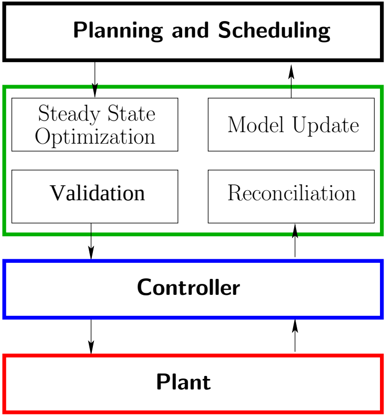
# State estimation




$$\min_{x_0, w(t)} \int_{-T}^{0} |y - g(x,u)|_R^2 + |\dot{x} - f(x,u)|_Q^2 \, dt$$

$$\dot{x} = f(x,u) + w \quad \text{(process noise)}$$
$$y = g(x,u) + v \quad \text{(measurement noise)}$$

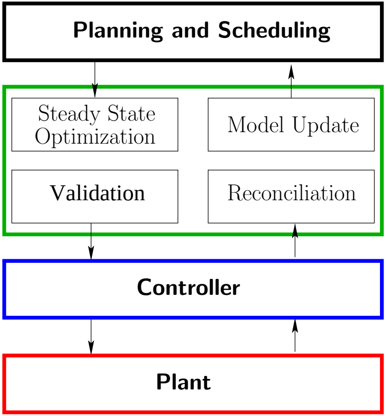
# Industrial practice of MPC




Two layer structure

- Steady-state layer
  - RTO optimizes steady state model
  - Optimal setpoints passed to dynamic layer

# Industrial practice of MPC




Two layer structure

- Steady-state layer
  - RTO optimizes steady state model
  - Optimal setpoints passed to dynamic layer
- Dynamic layer
  - Controller tracks the setpoints
  - Linear MPC (replaces multiloop PID)

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

## Eastman Chemical experience with MPC

- First MPC implemented in 1996

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

## Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

## Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity
- 30-50 M$/year increased profit due to increased throughput (2008)

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

## Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity
- 30-50 M$/year increased profit due to increased throughput (2008)

## Praxair experience with MPC

- Praxair currently has more than 150 MPC installations

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

## Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity
- 30-50 M$/year increased profit due to increased throughput (2008)

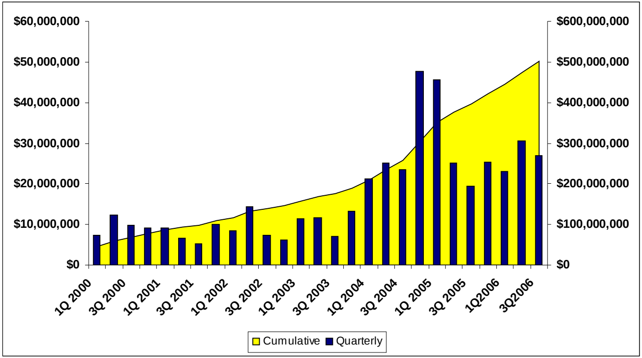
## Praxair experience with MPC

- Praxair currently has more than 150 MPC installations
- 16 M$/year increased profit (2008)

# Impact for 13 ethylene plants (Starks and Arrieta, 2007)

Dow
*Hydrocarbons AC&O*
17

## We're Doing it For the Money

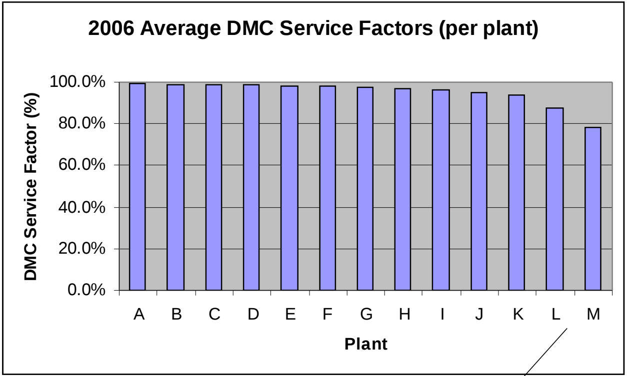



Advanced Control & Optimization

# Maintaining technology (Starks and Arrieta, 2007)



## Challenges (continued 1)




Plants L & M experienced APC personnel changes

# Broader industrial impact (Qin and Badgwell, 2003)

| Area | Aspen Technology | Honeywell Hi-Spec | Adersa | PCL | MDC | Total |
|---|---|---|---|---|---|---|
| Refining | 1200 | 480 | 280 | 25 | | 1985 |
| Petrochemicals | 450 | 80 | - | 20 | | 550 |
| Chemicals | 100 | 20 | 3 | 21 | | 144 |
| Pulp and Paper | 18 | 50 | - | - | | 68 |
| Air & Gas | - | 10 | - | - | | 10 |
| Utility | - | 10 | - | 4 | | 14 |
| Mining/Metallurgy | 8 | 6 | 7 | 16 | | 37 |
| Food Processing | - | - | 41 | 10 | | 51 |
| Polymer | 17 | - | - | - | | 17 |
| Furnaces | - | - | 42 | 3 | | 45 |
| Aerospace/Defense | - | - | 13 | - | | 13 |
| Automotive | - | - | 7 | - | | 7 |
| Unclassified | 40 | 40 | 1045 | 26 | 450 | 1601 |
| Total | 1833 | 696 | 1438 | 125 | 450 | 4542 |
| First App. | DMC:1985<br>IDCOM-M:1987<br>OPC:1987 | PCT:1984<br>RMPCT:1991 | IDCOM:1973<br>HIECON:1986 | PCL:<br>1984 | SMOC:<br>1988 | |
| Largest App | 603x283 | 225x85 | - | 31x12 | - | |

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?
- Is the technology mature?

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?
- Is the technology mature?
- Is the theory complete?

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?
- Is the technology mature?
- Is the theory complete?
- Do we have tools to decompose *large-scale systems* into manageable problems?

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?
- Is the technology mature?
- Is the theory complete?
- Do we have tools to decompose *large-scale systems* into manageable problems?
- Do we have tools to optimize dynamic *economic* operation?

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?
- Is the technology mature?
- Is the theory complete?
- Do we have tools to decompose *large-scale systems* into manageable problems?
- Do we have tools to optimize dynamic *economic* operation?
- Do we have tools to *commission* and *maintain* the controllers?

# Are all the problems solved?

## Some questions to consider

- Has the application base stopped growing?
- Is the technology mature?
- Is the theory complete?
- Do we have tools to decompose *large-scale systems* into manageable problems?
- Do we have tools to optimize dynamic *economic* operation?
- Do we have tools to *commission* and *maintain* the controllers?
- Can we handle *fast* systems?

# Has the application base stopped growing?

**European Commission**

**DG information Society & Media**

**Monitoring and control: today's market, its evolution till 2020 and the impact of ICT on these**

Workshop:

9th of October 2008

*A report & a presentation prepared by:*

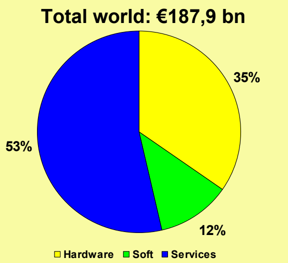
DECISION Etudes & Conseil

RPA

*Available for download: http://www.decision.eu/smart2007.htm*

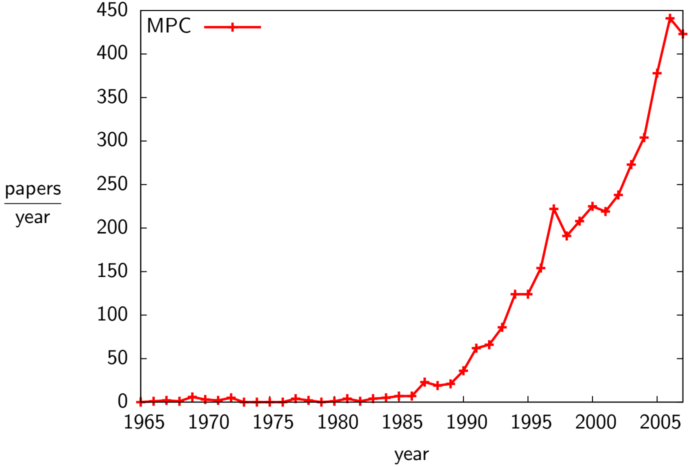
# 3. Worldwide Monitoring & Control Market

- The worldwide market for Monitoring & Control products and solutions is around 188 billion euros.
- This represents 8% of total ICT expenditures worldwide.
- In the field of ICT, this is comparable to:
  - the whole semiconductor industry world revenues;
  - twice the world mobile phone manufacturers revenues.
- **Services**, with **more than 50% of the market value**, have the biggest share.
- The 3 larger sub markets represent together over 100 billion euros, namely:
  - integration, installation & training services with 38 billion euros;
  - control hardware with 36 billion euros;
  - maintenance, repair & overall services with 30 billion euros.
- The 3 larger application markets are Vehicles, Process and Manufacturing industries.
- **Europe represents 32 % of the world total market value.**

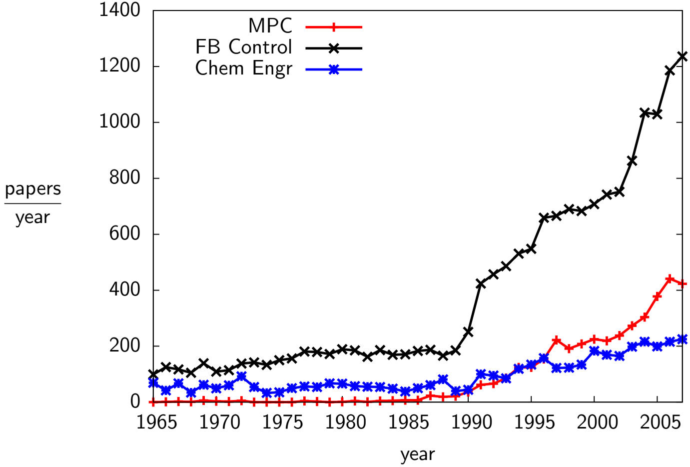



*Definitions*
- bundle OS & drivers included in hardware
- application development included in services

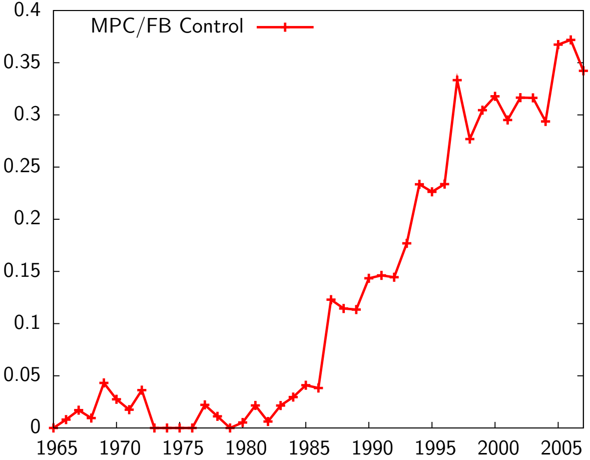
# Is the theory complete?



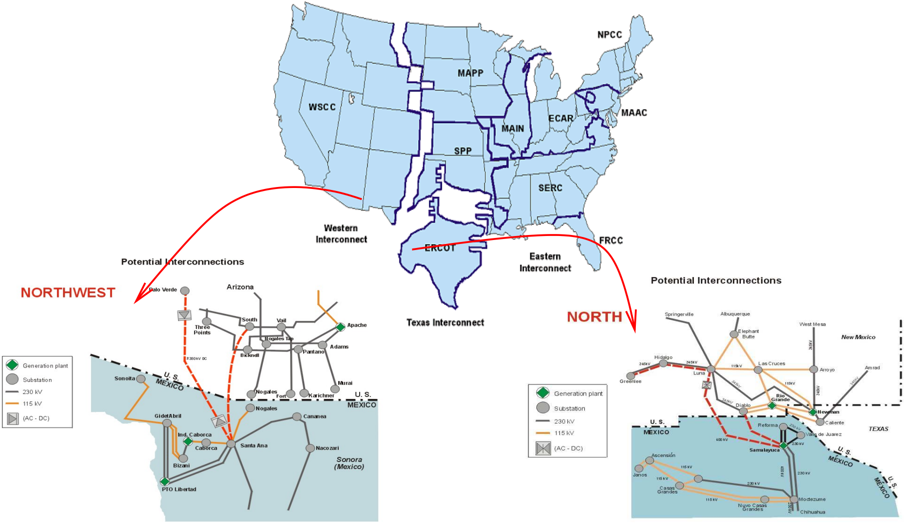
# Is the theory complete?



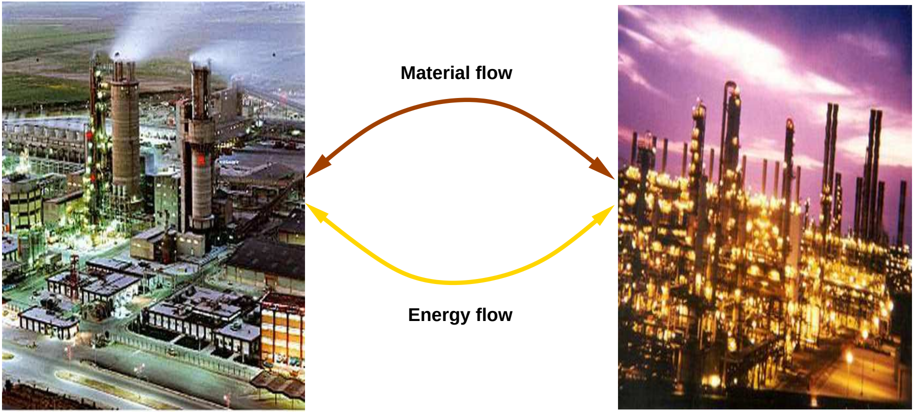
# Ratio of MPC papers to feedback control papers



# Distributed MPC — Decomposing large-scale systems



# Decomposing large-scale systems



# MPC at the large scale

## Decentralized Control

- Traditional approach
  - Wealth of literature from the early 1970's on improved decentralized control [a]
  - Well-known that poor performance may result if the interconnections are not negligible

[a] (Sandell Jr. et al., 1978; Šiljak, 1991; Lunze, 1992)

# MPC at the large scale

## Decentralized Control

- Traditional approach
  - Wealth of literature from the early 1970's on improved decentralized control [a]
  - Well-known that poor performance may result if the interconnections are not negligible

[a](Sandell Jr. et al., 1978; Šiljak, 1991; Lunze, 1992)

## Centralized Control

- Steady increase in available computational power has provided the opportunity for centralized control
- Many practitioners view centralized control of large, networked systems as impractical and unrealistic

# Cooperative control as suboptimal MPC

Properties established by suboptimal MPC theory[1]

- Stability: Given a feasible initial condition, adding the stability constraint

$$\|\mathbf{u}_i\| \leq d_i \|x_i(0)\|$$

  gives nominal closed-loop stability for any number of information exchanges

[1](Venkat et al., 2006a) and (Venkat et al., 2006b)

# Cooperative control as suboptimal MPC

Properties established by suboptimal MPC theory[1]

- Stability: Given a feasible initial condition, adding the stability constraint

$$\|\mathbf{u}_i\| \leq d_i \|x_i(0)\|$$

  gives nominal closed-loop stability for any number of information exchanges
- Cost decrease: Plant-wide objective is decreased at each iterate

[1] (Venkat et al., 2006a) and (Venkat et al., 2006b)

# Cooperative control as suboptimal MPC

Properties established by suboptimal MPC theory[1]

- Stability: Given a feasible initial condition, adding the stability constraint

$$\|\mathbf{u}_i\| \leq d_i \|x_i(0)\|$$

  gives nominal closed-loop stability for any number of information exchanges
- Cost decrease: Plant-wide objective is decreased at each iterate
- Convergence: Cooperative MPC produces centralized control performance at the limit of infinite iterates

[1] (Venkat et al., 2006a) and (Venkat et al., 2006b)

# Distributed MPC publications


G. Pannocchia, S. J. Wright, B. T. Stewart, and J. B. Rawlings. Efficient cooperative distributed MPC using partial enumeration. In *ADCHEM 2009, International Symposium on Advanced Control of Chemical Processes*, Istanbul, Turkey, July 12-15, 2009.

J. B. Rawlings and B. T. Stewart. Coordinating multiple optimization-based controllers: new opportunities and challenges. *J. Proc. Cont.*, 18:839–845, 2008.

B. T. Stewart, A. N. Venkat, J. B. Rawlings, S. J. Wright, and G. Pannocchia. Cooperative distributed model predictive control. Submitted for publication in *Sys. Cont. Let.*, July 2009.

A. N. Venkat, J. B. Rawlings, and S. J. Wright. Stability and optimality of distributed, linear MPC. Part 1: state feedback. Technical Report 2006–03, TWMCC, Department of Chemical and Biological Engineering, University of Wisconsin–Madison (Available at http://jbrwww.che.wisc.edu/tech-reports.html), October 2006a.

A. N. Venkat, J. B. Rawlings, and S. J. Wright. Stability and optimality of distributed, linear MPC. Part 2: output feedback. Technical Report 2006–04, TWMCC, Department of Chemical and Biological Engineering, University of Wisconsin–Madison (Available at http://jbrwww.che.wisc.edu/tech-reports.html), October 2006b.

# Large-scale systems and partial enumeration

- Unconstrained solution: LQ regulator (Kalman, 1960)

$$u = Kx$$

- Constrained solution: MPC

$$u_0 = K_i x + b_i$$

in which $i$ enumerates different possible active sets for the inequality constraints (Bemporad et al., 2002)

- There are $3^{mN}$ different active sets

$$\begin{bmatrix} \underline{u} \\ \underline{u} \\ \vdots \\ \underline{u} \end{bmatrix} \leq \begin{bmatrix} u^1 \\ u^2 \\ \vdots \\ u^m \end{bmatrix} \leq \begin{bmatrix} \overline{u} \\ \overline{u} \\ \vdots \\ \overline{u} \end{bmatrix}$$

# The active set table

$$u_0 = K_i x + b_i$$

# The active set table

$$u_0 = K_i x + b_i$$

$$N = 2$$

| $i$ | constraint set | $K_i$ | $b_i$ |
|---|---|---|---|
| 1 | $\{\overline{u}, \overline{u}\}$ | 0 | $\overline{u}$ |
| 2 | $\{\overline{u}, -\}$ | 0 | $\overline{u}$ |
| 3 | $\{\overline{u}, \underline{u}\}$ | 0 | $\overline{u}$ |
| 4 | $\{-, \overline{u}\}$ | $K_4$ | $b_4$ |
| 5 | $\{-, -\}$ | $K_5$ | $b_5$ |
| 6 | $\{-, \underline{u}\}$ | $K_6$ | $b_6$ |
| 7 | $\{\underline{u}, \overline{u}\}$ | 0 | $\underline{u}$ |
| 8 | $\{\underline{u}, -\}$ | 0 | $\underline{u}$ |
| 9 | $\{\underline{u}, \underline{u}\}$ | 0 | $\underline{u}$ |

# The active set table

$$u_0 = K_i x + b_i$$

$N = 2$

| $i$ | constraint set | $K_i$ | $b_i$ |
|---|---|---|---|
| 1 | $\{\overline{u}, \overline{u}\}$ | 0 | $\overline{u}$ |
| 2 | $\{\overline{u}, -\}$ | 0 | $\overline{u}$ |
| 3 | $\{\overline{u}, \underline{u}\}$ | 0 | $\overline{u}$ |
| 4 | $\{-, \overline{u}\}$ | $K_4$ | $b_4$ |
| 5 | $\{-, -\}$ | $K_5$ | $b_5$ |
| 6 | $\{-, \underline{u}\}$ | $K_6$ | $b_6$ |
| 7 | $\{\underline{u}, \overline{u}\}$ | 0 | $\underline{u}$ |
| 8 | $\{\underline{u}, -\}$ | 0 | $\underline{u}$ |
| 9 | $\{\underline{u}, \underline{u}\}$ | 0 | $\underline{u}$ |

$N = 4$

| $i$ | constraint set | $K_i$ | $b_i$ |
|---|---|---|---|
| 1 | $\{\overline{u}, \overline{u}, \overline{u}, \overline{u}\}$ | 0 | $\overline{u}$ |
| 2 | $\{\overline{u}, \overline{u}, \overline{u}, -\}$ | 0 | $\overline{u}$ |
| 3 | $\{\overline{u}, \overline{u}, \overline{u}, \underline{u}\}$ | 0 | $\overline{u}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| 40 | $\{-, -, -, \overline{u}\}$ | $K_{40}$ | $b_{40}$ |
| 41 | $\{-, -, -, -\}$ | $K_{41}$ | $b_{41}$ |
| 42 | $\{-, -, -, \underline{u}\}$ | $K_{42}$ | $b_{42}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| 79 | $\{\underline{u}, \underline{u}, \underline{u}, \overline{u}\}$ | 0 | $\underline{u}$ |
| 80 | $\{\underline{u}, \underline{u}, \underline{u}, -\}$ | 0 | $\underline{u}$ |
| 81 | $\{\underline{u}, \underline{u}, \underline{u}, \underline{u}\}$ | 0 | $\underline{u}$ |

# The computational challenge of large scale *centralized* MPC

- On-line QP solution may not be practical for large-scale applications

# The computational challenge of large scale *centralized* MPC

- On-line QP solution may not be practical for large-scale applications
- Computation time may be too long

# The computational challenge of large scale *centralized* MPC

- On-line QP solution may not be practical for large-scale applications
- Computation time may be too long
- Full enumeration is not possible!

# The computational challenge of large scale *centralized* MPC

- On-line QP solution may not be practical for large-scale applications
- Computation time may be too long
- Full enumeration is not possible!
- Typical large-sized applications:

$$m = 5, \quad N = 50, \quad 10^{119}$$

$$m = 32, \quad N = 15, \quad 10^{229}$$

# The computational challenge of large scale *centralized* MPC

- On-line QP solution may not be practical for large-scale applications
- Computation time may be too long
- Full enumeration is not possible!
- Typical large-sized applications:

$$m = 5, \quad N = 50, \quad 10^{119}$$

$$m = 32, \quad N = 15, \quad 10^{229}$$

- Consider partial enumeration

# Partial Enumeration

Conjecture: In practice, the state might visit relatively few different active constraints! **SO...** don't do the calculations for all possible active sets. Instead:

- Initialize by calculating and storing the table only for "likely" active sets;
- Given $\hat{x}$, search through the table and see if one of them is optimal;
- If so, use that control;
- If not, is there a feasible (suboptimal) solution? If so, use this one;
- If not, solve the MPC QP problem with small $N$ and add this solution to the table;
- Make room in the table by deleting the entry that was used least recently.

# Initializing the Table

- Choose a size for the table, and initialize its contents in a training period.
- Add disturbances during training period to ensure that "useful" parts of the active-set space are sampled.
- Larger disturbances tend to cause saturation of some inputs.
- Saturation causes more active sets to be "interesting" because of small variations in the timepoint at which a constraint becomes active/inactive.

# Computational Results: Problem 1

- Copolymerization reactor: 5 inputs, 18 states, 4 outputs, horizon $N = 50$.
- $10^{119}$ possible active sets.

# Computational Results: Problem 1

- Copolymerization reactor: 5 inputs, 18 states, 4 outputs, horizon $N = 50$.
- $10^{119}$ possible active sets.
- Training period: 28800 sample intervals (2 days, 1-minute intervals), 20 random disturbances applied. Visited a total of 376 active sets during this time.
- Validation period: 43200 sample intervals (3 days), 30 random disturbances.
- Considered tables of sizes 50, 100, and 200.

# CPU time and suboptimality ratio of four solvers

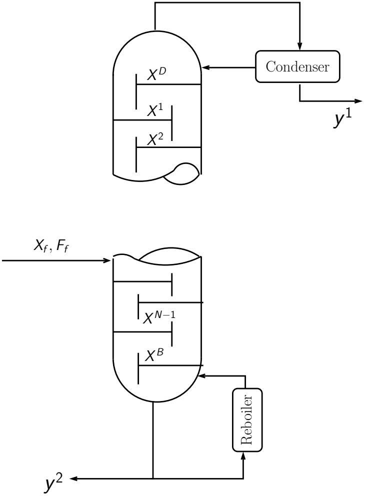
| | TAB50 | TAB100 | TAB200 | `qpsol` |
|---|---|---|---|---|
| Max time (s) | 0.144 | 0.278 | 0.463 | 1.56 |
| Mean time (s) | 0.0075 | 0.0098 | 0.0129 | 1.02 |
| Active sets visited | 771 | 814 | 877 | – |
| Suboptimality | 1.0090 | 1.0088 | 1.0094 | – |

# Partial enumeration publications


G. Pannocchia, J. B. Rawlings, and S. J. Wright. Fast, large-scale model predictive control by partial enumeration. *Automatica*, 43:852–860, 2007.

G. Pannocchia, S. J. Wright, B. T. Stewart, and J. B. Rawlings. Efficient cooperative distributed MPC using partial enumeration. In *ADCHEM 2009, International Symposium on Advanced Control of Chemical Processes*, Istanbul, Turkey, July 12-15, 2009.

# Controller Commissioning — Obtaining $Q$ and $R$ from Data



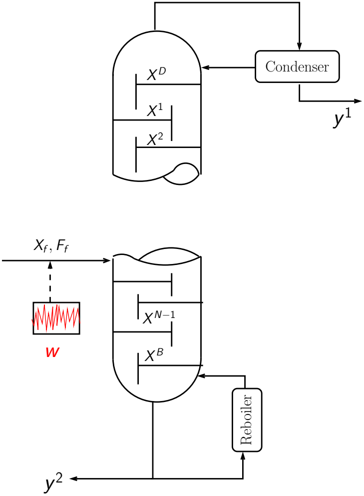

- Model discretized with $t_k = k\Delta t$:

$$\frac{d}{dt}\underbrace{\begin{bmatrix} X^D \\ \vdots \\ X^B \end{bmatrix}}_{x(t)} = F(x(t), \underbrace{u(t)}_{X_f, F_f})$$

$$\begin{bmatrix} y^1 \\ y^2 \end{bmatrix}(t_k) = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} x(t_k)$$

- Measurements are only $X^D, X^B$ at the discretization times

# Controller Commissioning — Obtaining $Q$ and $R$ from Data



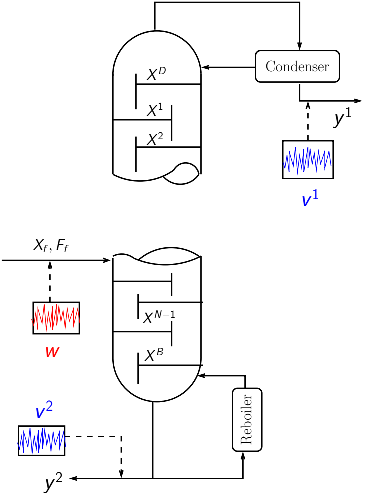

- Model discretized with $t_k = k\Delta t$:

$$x_{k+1} = f(x_k, u_k) + g(x_k, u_k) w_k$$

$$\begin{bmatrix} y^1 \\ y^2 \end{bmatrix}_k = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} x_k$$

- Measurements are only $X^D, X^B$ at the discretization times
- Noise $w_k$ affects all the states

# Controller Commissioning — Obtaining $Q$ and $R$ from Data



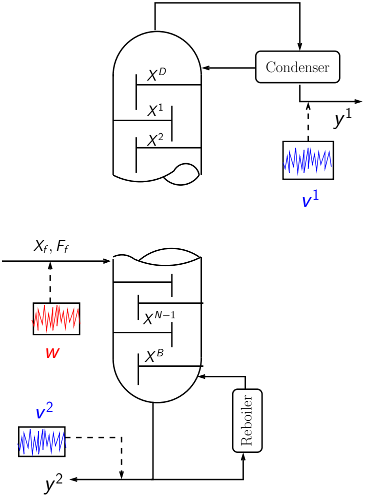

- Model discretized with $t_k = k\Delta t$:

$$x_{k+1} = f(x_k, u_k) + g(x_k, u_k) w_k$$

$$\begin{bmatrix} y^1 \\ y^2 \end{bmatrix}_k = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} x_k + \begin{bmatrix} v^1 \\ v^2 \end{bmatrix}_k$$

- Measurements are only $X^D, X^B$ at the discretization times
- Noise $w_k$ affects all the states
- Noise $v_k$ corrupts the measurements

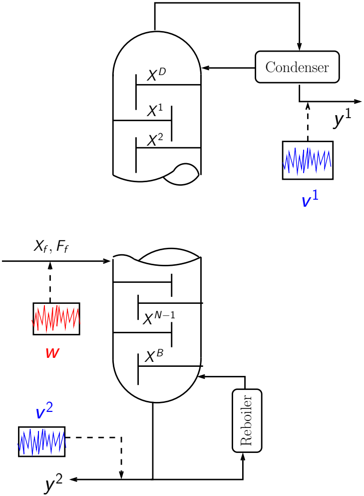
# Motivation for Using Autocovariances




## Idea of Autocovariances

- The state noise $w_k$ gets propagated in time
- The measurement noise $v_k$ appears only at the sampling times and is not propagated in time
- Taking autocovariances of data at different time lags gives covariances of $w_k$ and $v_k$

# Motivation for Using Autocovariances




## Idea of Autocovariances

- The state noise $w_k$ gets propagated in time
- The measurement noise $v_k$ appears only at the sampling times and is not propagated in time
- Taking autocovariances of data at different time lags gives covariances of $w_k$ and $v_k$

Let $w_k$, $v_k$ have zero means and covariances $Q$ and $R$

# Mathematical Formulation of the ALS

## Linear State-Space Model:

$$x_{k+1} = Ax_k + Gw_k \qquad w_k \sim N(0, Q)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R)$$

- Model $(A, C, G)$ known from the linearization, finite set of measurements: $\{y_0, \ldots, y_k\}$ given.
- Only unknowns are noises $w_k$ and $v_k$.

# Mathematical Formulation of the ALS

**Linear State-Space Model:**

$$x_{k+1} = Ax_k + Gw_k \qquad w_k \sim N(0, Q)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R)$$

- Model $(A, C, G)$ known from the linearization, finite set of measurements: $\{y_0, \ldots, y_k\}$ given.
- Only unknowns are noises $w_k$ and $v_k$.

- $y_k = Cx_k$
- $y_{k+1} = CAx_k + CGw_k$
- $y_{k+2} = CA^2x_k + CAGw_k + CGw_{k+1}$

# Mathematical Formulation of the ALS

**Linear State-Space Model:**

$$x_{k+1} = Ax_k + Gw_k \qquad w_k \sim N(0, Q)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R)$$

- Model $(A, C, G)$ known from the linearization, finite set of measurements: $\{y_0, \ldots, y_k\}$ given.
- Only unknowns are noises $w_k$ and $v_k$.

- $y_k = Cx_k + v_k$
- $y_{k+1} = CAx_k + CGw_k + v_{k+1}$
- $y_{k+2} = CA^2x_k + CAGw_k + CGw_{k+1} + v_{k+2}$

# Mathematical Formulation of the ALS

## Linear State-Space Model:

$$x_{k+1} = Ax_k + G w_k \qquad w_k \sim N(0, Q)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R)$$

- Model $(A, C, G)$ known from the linearization, finite set of measurements: $\{y_0, \ldots, y_k\}$ given.
- Only unknowns are noises $w_k$ and $v_k$.

- $y_k = Cx_k + v_k$
- $y_{k+1} = CAx_k + CG w_k + v_{k+1}$
- $y_{k+2} = CA^2 x_k + CAG w_k + CG w_{k+1} + v_{k+2}$

- $E[y_k y_k'] = R$

# Mathematical Formulation of the ALS

## Linear State-Space Model:

$$x_{k+1} = Ax_k + Gw_k \qquad w_k \sim N(0, Q)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R)$$

- Model $(A, C, G)$ known from the linearization, finite set of measurements: $\{y_0, \ldots, y_k\}$ given.
- Only unknowns are noises $w_k$ and $v_k$.

- $y_k = Cx_k + v_k$
- $y_{k+1} = CAx_k + CGw_k + v_{k+1}$
- $y_{k+2} = CA^2x_k + CAGw_k + CGw_{k+1} + v_{k+2}$

- $E[y_k y_k'] = R$
- $E[y_{k+2} y_{k+1}'] = CAGQG'C'$

# The Autocovariance Least-Squares (ALS) Problem

Skipping a lot of algebra, we can write:

**Autocovariance Least Squares**

$$\Phi = \min_{Q,R} \left\| \mathscr{A}_N \begin{bmatrix} (Q)_s \\ (R)_s \end{bmatrix} - \hat{b} \right\|^2$$

1. A least-squares problem in a vector of unknowns, $Q$, $R$

# The Autocovariance Least-Squares (ALS) Problem

Skipping a lot of algebra, we can write:

## Autocovariance Least Squares

$$\Phi = \min_{Q,R} \left\| \mathscr{A}_N \begin{bmatrix} (Q)_s \\ (R)_s \end{bmatrix} - \hat{b} \right\|^2$$

1. A least-squares problem in a vector of unknowns, $Q$, $R$
2. Form $\mathscr{A}_N$ from known system matrices

# The Autocovariance Least-Squares (ALS) Problem

Skipping a lot of algebra, we can write:

**Autocovariance Least Squares**

$$\Phi = \min_{Q,R} \left\| \mathscr{A}_N \begin{bmatrix} (Q)_s \\ (R)_s \end{bmatrix} - \hat{b} \right\|^2$$

1. A least-squares problem in a vector of unknowns, $Q$, $R$
2. Form $\mathscr{A}_N$ from known system matrices
3. $\hat{b}$ is a vector containing the estimated correlations from data

$$\hat{b} = \frac{1}{T} \sum_{k=1}^{\prime} \begin{bmatrix} y_k y_k' \\ \vdots \\ y_{k+N-1} y_k' \end{bmatrix}_s$$

# State estimation using $(Q, R)$ from ALS

- Efficient software developed for ALS

# State estimation using $(Q, R)$ from ALS

- Efficient software developed for ALS
- Data requirements for $(Q, R)$ identification are reasonable

# State estimation using $(Q, R)$ from ALS

- Efficient software developed for ALS
- Data requirements for $(Q, R)$ identification are reasonable
- Industrial testing underway

# State estimation using $(Q, R)$ from ALS

- Efficient software developed for ALS
- Data requirements for $(Q, R)$ identification are reasonable
- Industrial testing underway
- Preliminary results from Eastman chemical cracking furnace and Aspentech evaluation are encouraging

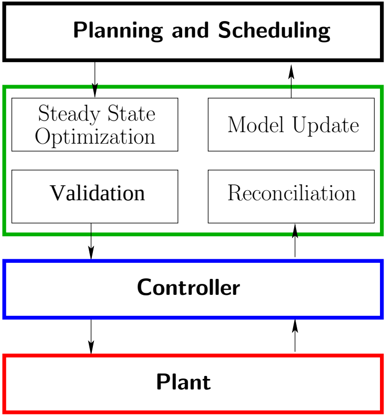
# Distributed MPC publications


B. J. Odelson, M. R. Rajamani, and J. B. Rawlings. A new autocovariance least-squares method for estimating noise covariances. *Automatica*, 42(2): 303–308, February 2006.

M. R. Rajamani and J. B. Rawlings. Estimation of the disturbance structure from data using semidefinite programming and optimal weighting. *Automatica*, 45: 142–148, 2009.

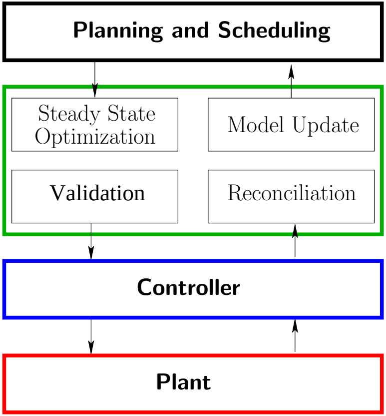
M. R. Rajamani, J. B. Rawlings, and S. J. Qin. Achieving state estimation equivalence for misassigned disturbances in offset-free model predictive control. *AIChE J.*, 55(2):396–407, February 2009.

# Optimizing economics: Current industrial practice



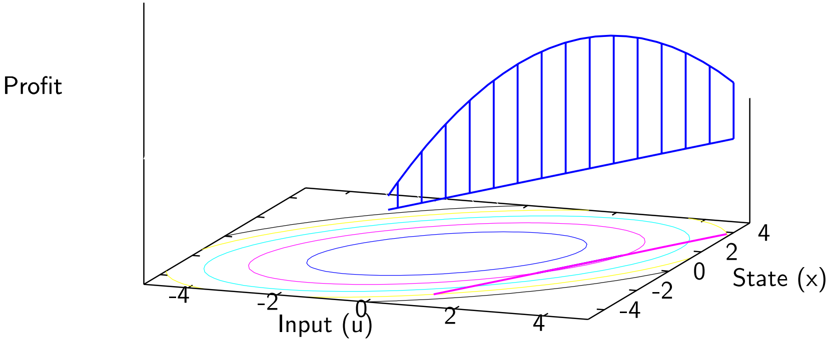

- Two layer structure
- Drawbacks

# Optimizing economics: Current industrial practice

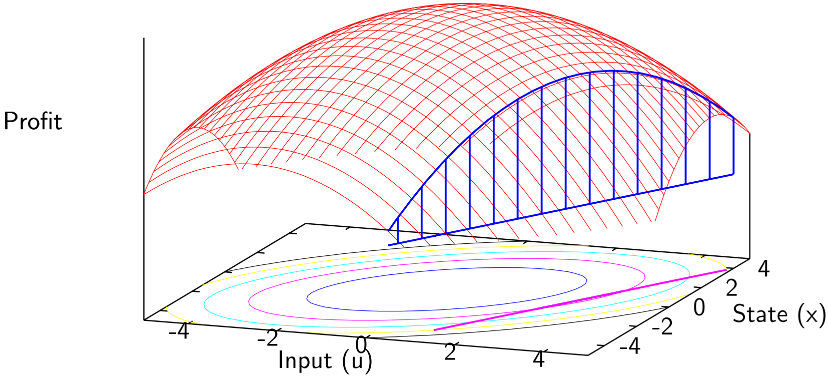



- Two layer structure
- Drawbacks
  - Inconsistent models
  - Re-identify linear model as setpoint changes
  - Time scale separation may not hold
  - Economics unavailable in dynamic layer

# Optimizing economics: what's desirable?



# Optimizing economics: what's desirable?



# Economics MPC publications


D. Angeli, R. Amrit, and J. B. Rawlings. Receding horizon cost optimization for overly constrained nonlinear plants. In *Proceedings of the Conference on Decision and Control*, Shanghai, China, December 2009.

J. B. Rawlings and R. Amrit. Optimizing process economic performance using model predictive control. In L. Magni, D. M. Raimondo, and F. Allgöwer, editors, *Nonlinear Model Predictive Control*, volume 384 of *Lecture Notes in Control and Information Sciences*, pages 119–138, Berlin, 2009. Springer.

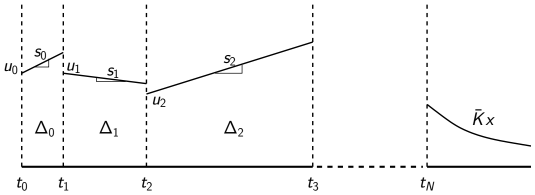
L. Würth, J. B. Rawlings, and W. Marquardt. Economic dynamic real-time optimization and nonlinear model-predictive control on infinite horizons. In *ADCHEM 2009, International Symposium on Advanced Control of Chemical Processes*, Istanbul, Turkey, July 12-15, 2009.

# Continuous-time system and input parameterization

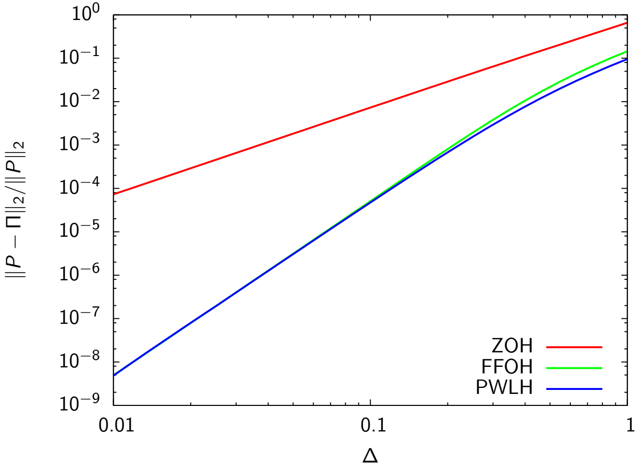
We consider linear time-invariant continuous-time systems

$$\dot{x} = Ax + Bu \tag{1}$$

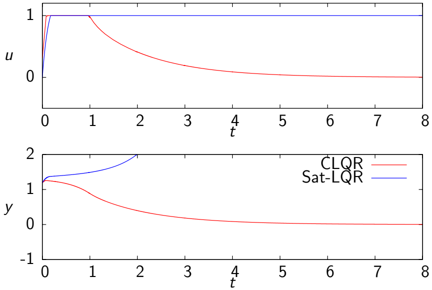
$$V(x_0, \mathbf{u}) = \frac{1}{2} \int_0^\infty \left( x' Q x + u' R u \right) dt \tag{2}$$

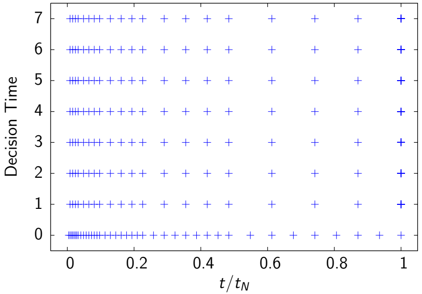


# Relative error between CT and DT systems

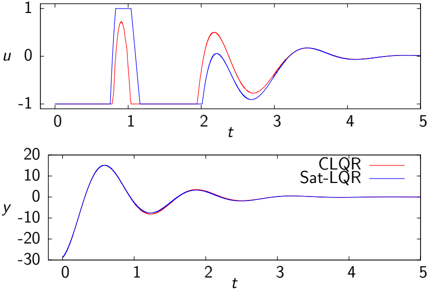


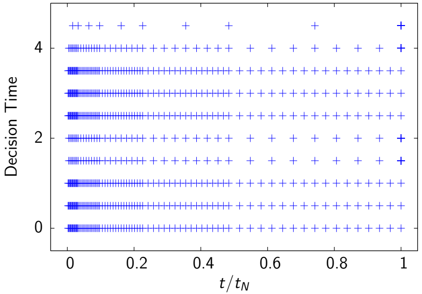
# Open-loop unstable system



# System with slow and fast modes



# Computation time for infinite horizon CT solution

| | FFOH-CLQR Algorithm | | |
|---|---|---|---|
| $\mu$ | $\frac{V^{\mathrm{II}}(x_0)-V^*(x_0)}{V^*(x_0)}$ | $N$ | CPU-time (ms) |
| $10^{-2}$ | $7.689 \cdot 10^{-4}$ | 12 | 0.20 |
| $10^{-3}$ | $1.883 \cdot 10^{-4}$ | 24 | 0.40 |
| $10^{-4}$ | $4.926 \cdot 10^{-8}$ | 96 | 7.5 |
| $10^{-5}$ | $4.926 \cdot 10^{-8}$ | 96 | 7.5 |
| $10^{-6}$ | $4.926 \cdot 10^{-8}$ | 96 | 7.5 |
| $10^{-7}$ | $8.841 \cdot 10^{-9}$ | 192 | 44.0 |
| $10^{-8}$ | $1.468 \cdot 10^{-9}$ | 384 | 444 |

# Continuous time MPC publications


G. Pannocchia, J. B. Rawlings, D. Q. Mayne, and W. Marquardt. On computing the solutions to the continuous time constrained linear quadratic regulator, July 2009a. Accepted for publication in IEEE Trans. Auto. Cont.

G. Pannocchia, J. B. Rawlings, D. Q. Mayne, and W. Marquardt. Computation of the infinite horizon continuous time constrained linear quadratic regulator. In *ADCHEM 2009, International Symposium on Advanced Control of Chemical Processes*, Istanbul, Turkey, July 12-15, 2009b.

# Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years.

# Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years. Both in theory and in practice.

# Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years. Both in theory and in practice.
- The currently available theory splits the problem into state estimation and regulation.

# Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years. Both in theory and in practice.
- The currently available theory splits the problem into state estimation and regulation. Both are posed and solved as online optimization problems.

# Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years. Both in theory and in practice.
- The currently available theory splits the problem into state estimation and regulation. Both are posed and solved as online optimization problems. Basic properties have been established.

# Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years. Both in theory and in practice.
- The currently available theory splits the problem into state estimation and regulation. Both are posed and solved as online optimization problems. Basic properties have been established.
- The economic impact of the technology is large.

# Future Outlook

1. Distributed MPC.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.
3. Controller commissioning.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.
3. Controller commissioning. ALS properties established and efficient prototype software developed. Has had some industrial testing.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.
3. Controller commissioning. ALS properties established and efficient prototype software developed. Has had some industrial testing.
4. Optimizing dynamic economic objective.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.
3. Controller commissioning. ALS properties established and efficient prototype software developed. Has had some industrial testing.
4. Optimizing dynamic economic objective. Early stages. Theory finished for linear models and convex cost functions. Extensions to nonlinear models and nonconvex cost functions under study.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.
3. Controller commissioning. ALS properties established and efficient prototype software developed. Has had some industrial testing.
4. Optimizing dynamic economic objective. Early stages. Theory finished for linear models and convex cost functions. Extensions to nonlinear models and nonconvex cost functions under study.
5. Continuous time MPC.

# Future Outlook

1. Distributed MPC. Basic properties of cooperative MPC have been established. Ready for industrial testing.
2. Partial enumeration. Tested on large examples. Ready for industrial testing.
3. Controller commissioning. ALS properties established and efficient prototype software developed. Has had some industrial testing.
4. Optimizing dynamic economic objective. Early stages. Theory finished for linear models and convex cost functions. Extensions to nonlinear models and nonconvex cost functions under study.
5. Continuous time MPC. Algorithm developed and tested on examples. Theory for convergence and closed-loop stability under development now.

# Acknowledgments


- Don Bartusiak, ExxonMobil
- Tom Badgwell, Aspentech
- Jim Downs, Eastman Chemical
- Larry Megan, Praxair
- Rahul Bindlish, Dow
- Financial support from NSF #CTS-0825306, 0931835 and Texas Wisconsin California Control Consortium (TWCCC) members

# Further reading


A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos. The explicit linear quadratic regulator for constrained systems. *Automatica*, 38(1):3–20, 2002.

R. E. Kalman. Contributions to the theory of optimal control. *Bull. Soc. Math. Mex.*, 5:102–119, 1960.

J. Lunze. *Feedback Control of Large Scale Systems*. Prentice-Hall, London, U.K., 1992.

S. J. Qin and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Eng. Prac.*, 11(7):733–764, 2003.

N. R. Sandell Jr., P. Varaiya, M. Athans, and M. Safonov. Survey of decentralized control methods for larger scale systems. *IEEE Trans. Auto. Cont.*, 23(2):108–128, 1978.

D. D. Šiljak. *Decentralized Control of Complex Systems*. Academic Press, London, 1991. ISBN 0-12-643430-1.

D. M. Starks and E. Arrieta. Maintaining AC&O applications, sustaining the gain. In *Proceedings of National AIChE Spring Meeting*, Houston, Texas, April 2007.