

## Modeling and Analysis Principles for Chemical and Biological Engineers

### Errata for First Printing

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May 15, 2018

1. Page 5. Change  $y_n$  to  $y_m$  in second to last displayed equation. Thanks to Nishith Patel of UW for pointing out this erratum.
2. Pages 7–8. In Example 1.4, matrix  $A$  is missing a minus sign. Replace  $A$ ,  $AB$  and  $BA$  by the following:
 
$$A = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \quad AB = \begin{bmatrix} \sqrt{2} & -\frac{1}{2\sqrt{2}} \\ \sqrt{2} & \frac{1}{2\sqrt{2}} \end{bmatrix} \quad BA = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\sqrt{2} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$
- Thanks to Shu Pan of UW for pointing out this erratum.
3. Page 12, line 13. Replace “outer project” with “outer product.” Thanks to Ted Walker of UW for pointing out this erratum.
4. Page 15, first line change  $b \in \mathbb{R}^n$  to  $b \in \mathbb{R}^m$ . Thanks to Apoorva Sampat of UW for pointing out this erratum.
5. Page 37, 13 lines from bottom. Replace  $Av_j = \lambda_j$  with  $Av_j = \lambda_j v_j$ . Thanks to Nishith Patel of UW for pointing out this erratum.
6. Page 45. In second displayed equation, replace  $e^{-\omega t}$  with  $e^{-i\omega t}$ . Thanks to Nishith Patel of UW for pointing out this erratum.
7. Page 49. In fifth displayed equation, replace second  $S$  with  $S^*$ , and  $S^2$  with  $SS^*$ . In sixth displayed equation, replace first  $S$  with  $S^*$ , and  $S^2$  with  $S^*S$ .
8. Page 49, six lines from bottom. Replace  $m - n$  with  $n - m$ . Page 50, four lines from bottom. Replace  $n - m$  with  $m - n$ . Thanks to Ralph White of Univ. of South Carolina for pointing out this erratum.
9. Page 52, line after first displayed equation in Section 1.5.1. Change  $i = 1, \dots, n$  to  $i = 0, \dots, n$ . Thanks to Sanjan Gupta of UW for pointing out this erratum.
10. Page 55. Eighth line of the last full paragraph: replace “More generally,  $\Lambda^{1/2}$  can be replaced by  $Q^* \Lambda^{1/2} Q$  for any unitary matrix  $Q$ ” with “More generally,  $\Lambda^{1/2}$  can be replaced by  $\text{diag}(\pm\lambda_1^{1/2}, \pm\lambda_2^{1/2}, \dots)$ .” Thanks to Coogan Thompson of UW for pointing out this issue.
11. Page 60, second to last line of first paragraph. Change  $A > 0$  to  $A < 0$ . Thanks to Sanjan Gupta of UW for pointing out this erratum.
12. Page 68, first displayed equation. Change  $\sigma(A \otimes B)$ ,  $\sigma(A)$ , and  $\sigma(B)$ , to  $\sigma^2(A \otimes B)$ ,  $\sigma^2(A)$ , and  $\sigma^2(B)$ , respectively. Thanks to Sanjan Gupta of UW for pointing out this erratum.
13. Page 95. For improved clarity, add the following to the end of Exercise 1.77 (d).

Note that for a matrix (like  $A$ ) with linearly independent columns, the pseudoinverse is  $A^\dagger = (A^T A)^{-1} A^T$ , but for a matrix (like  $B$ ) with linearly independent rows, the pseudoinverse is  $B^\dagger = B^T (B B^T)^{-1}$ . The matrices  $A^\dagger$  and  $B^\dagger$  are also known as left and right inverses, respectively.

Thanks to Megan Zagrobelny of UW for pointing out this issue.

14. Page 100. In the first equation, change  $A^2 t$  to  $A^2 t^2$ , and change  $A^3 t$  to  $A^3 t^3$ .
15. Page 100. In the second equation change  $S$  to  $M$ . (Though in the case under consideration, where  $A$  is diagonalizable,  $M$  reduces to  $S$  as described in Chapter 1.)
16. Page 100. In the last displayed equation, change  $\text{Re}(c_1 e^{\sigma t} e^{i\omega t} v)$  to  $2\text{Re}(c_1 e^{\sigma t} e^{i\omega t} v)$ . Thanks to Sanjan Gupta of UW for pointing out this erratum.
17. Page 101, first displayed equation. Change the sign on the 2nd and 4th terms of the sum on the right-hand side. Thanks to Sanjan Gupta of UW for pointing out this erratum.
18. Page 107, Table 2.1. Change  $b$  to  $\omega$  in last two entries. Thanks to Harmon Ray of UW for pointing out this erratum.
19. Page 109. Fourth line in the solution of Example 2.2. Replace  $s + a$  with  $s - a$  in the denominator of the first term on the right-hand side of the equation.
20. Page 114. In the first full paragraph, “ $a_0 = 1, a_1 = -2, a_2 = 1$ ” should read “ $a_0 = 1, a_1 = -1, a_2 = 1$ ”. Thanks to Ralph White of Univ. of South Carolina for pointing out this erratum.
21. Page 171. Fourth line in part (c). Change  $t_f = t/k_{-1}$  to  $t_f = k_{-1} t$ . Thanks to Luo Ji of UW for pointing out this erratum.
22. Page 226, Exercise 2.20(b). The equation  $\omega_1^2 + \omega_2^2 + \omega_3^2 = 1$  should be replaced by  $I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + I_3^2 \omega_3^2 = 1$ .
23. Page 235, Exercise 2.44(b). Change  $y(x)$  to  $u(x)$ .
24. Page 237, Exercise 2.49. For consistency with the text, vectors  $x_n, y_{n+M}$  etc. should be rewritten  $x^{(n)}, y^{(n+M)}$  etc. Also, change  $A$  to  $G$  in part (b).
25. Page 237, Exercises 2.50 and 2.51. For consistency with the text, vectors  $x_n, u_n, v_n$  etc. should be rewritten  $x^{(n)}, u^{(n)}, v^{(n)}$  etc.
26. Page 239, Exercise 2.54. This exercise has been updated. It should read

The eigenvalue problem

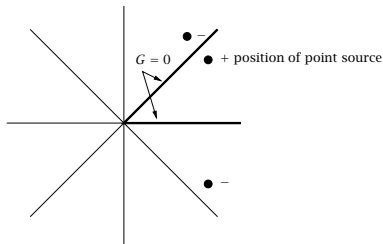
$$y'' + (x + \lambda)y = 0$$

arises in optics, quantum mechanics, and hydrodynamics, and is known as Airy’s equation.

- (a) If only an estimate of the smallest (magnitude) eigenvalue  $\lambda$  is needed, a *one-term* Galerkin approximation can be useful. In quantum mechanics, where eigenvalues have the interpretation of energy levels, this approach is a version of the GROUND STATE APPROXIMATION. Use a one-term Galerkin approximation using a sine function basis,  $\phi_j = \sin j\pi x$ , to estimate the first eigenvalue of Airy’s equation with boundary conditions  $y(0) = y(1) = 0$ . That is, let  $y(x) \approx c_1 \sin \pi x$ .

- (b) Repeat with a two-term Galerkin approximation,  $y(x) \approx c_1 \sin \pi x + c_2 \sin 2\pi x$ .
- (c) Use the finite element method with hat functions to construct an *algebraic* problem for the Airy equation: it will have the structure  $Au + \lambda Bu = 0$ . The matrix  $B$  will be invertible so this form can be converted to a standard algebraic eigenvalue problem. Find the approximate eigenvalues and eigenfunctions using six hat functions and again with 12.

- 27. Page 260, Figure 3.2. Change  $\delta_r, \delta_\theta$  to  $e_r, e_\theta$ .
- 28. Page 263, Figure 3.3. Change  $\delta_r, \delta_\theta, \delta_\phi$  to  $e_r, e_\theta, e_\phi$ .
- 29. Page 268, 4th displayed equation. Remove the extraneous  $dV$  in the  $\nabla \cdot F_A dV$  term in brackets. Thanks to Sanjan Gupta of UW for pointing out this erratum.
- 30. Page 279, Example 3.6. Three sign changes in last sentence: change  $w_{xx} + w_{yy} = \lambda w$  to  $w_{xx} + w_{yy} = -\lambda w$ , and  $\lambda_{mn} u_{mn} = f_{mn}$  to  $-\lambda_{mn} u_{mn} = f_{mn}$ , and  $u_{mn} = f_{mn}/\lambda_{mn}$  to  $u_{mn} = -f_{mn}/\lambda_{mn}$ . Also, in the final equation of the example, change the index in the second summation from  $m$  to  $n$ .
- 31. Page 282, last three equations. In the arguments of the modified Bessel functions  $I_0$  and  $K_0$ , replace  $n^2 \pi^2$  by  $n\pi$ .
- 32. Page 309, first displayed equation. In the denominator of  $a_{ni}$ , change  $(r_n - 1)!$  to  $(r_n - i)!$ . The same erratum is on page 530 below.
- 33. Page 314, last displayed equation. Remove both leading minus signs in the two cases of the  $\bar{G}(x, \xi, s)$  expression. Thanks to Ahmed Elnabawy of UW for pointing out this and the next erratum.
- 34. Page 315, fourth displayed equation from the bottom. Replace  $(-1)^n$  with  $(-1)^{n+1}$  in the two cases of the  $G(x, \xi, \tau)$  expression.
- 35. Page 325, Exercise 3.1, line 10. Change  $x_1 = 1$  to  $x_1 = x$ . Delete the extraneous sentence starting "The same arguments hold ..." to the end of the exercise.
- 36. Page 339, Exercise 3.42. The images in the figure are not shown in a format consistent with the text. The correct figure appears as follows.



- 37. Page 343, Exercise 3.52. Replace "AM2" with "implicit Euler".

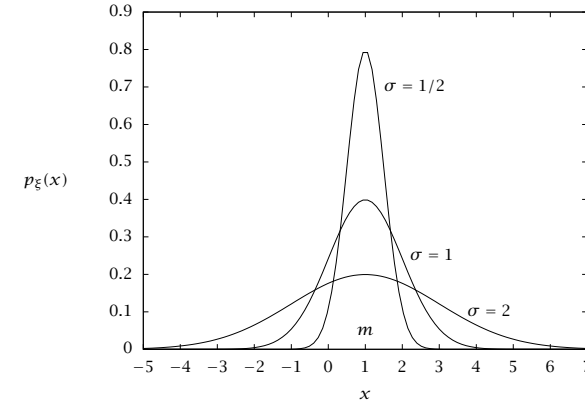
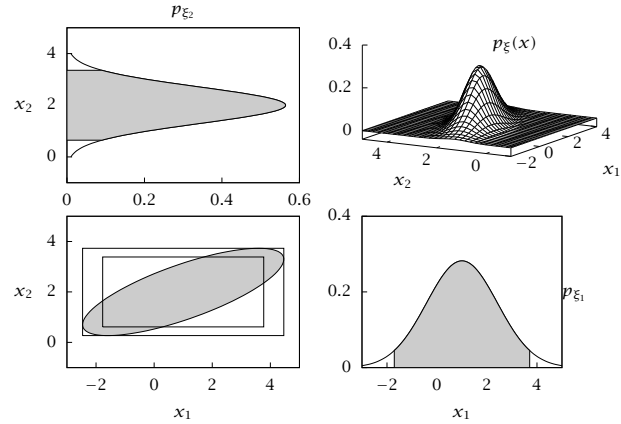


Figure 4.1: Normal distribution, with probability density  $p_\xi(x) = (1/\sqrt{2\pi\sigma^2}) \exp(-(1/2)(x-m)^2/\sigma^2)$ . Mean is one and standard deviations are 1/2, 1, and 2.

- 38. Page 343, Exercise 3.53. Replace "Adams-Moulton" with "implicit Euler", " $\alpha = 0.05$ " with " $\alpha = 0.1$ " and " $T(-1, 0) = 1$ " with " $T(-1, t) = 1$ ".
- 39. Pages 343 and 345. Replace "Levy" with "Lewy".
- 40. Pages 344, 348 and 568. Replace "Nicholson" with "Nicolson".
- 41. Page 355, Figure 4.1 should appear as shown here. Note the  $\sigma^2$  labels are changed to  $\sigma$  in the figure (three places), and "variances" is changed to "standard deviations" in the caption.
- 42. Page 356, second displayed equation. Change  $e^{-itx}$  to  $e^{itx}$ . In the third displayed equation, change  $e^{-itz}$  to  $e^{itz}$ . Thanks to Ralph White of Univ. of South Carolina for pointing out this erratum.
- 43. Page 363, first displayed equation. Put absolute value around  $\det Q$ .
- 44. Page 377, Ninth line from top, "...and there is no random component  $\xi_2$ ." Change to "...and there is no component  $\xi_1$ ." Thanks to Travis Arnold of UW for pointing out this erratum.
- 45. Page 385, fourth displayed equation. Change  $i\mathcal{E}(Y_i)$  term to  $i\mathcal{E}(Y_i)$ . Thanks to Travis Arnold of UW for pointing out this erratum.
- 46. Page 389, fifth displayed equation. Change  $p_{\eta, \xi}$  on the left-hand side of the equality to  $p_{\eta|\xi}$ . Thanks to Travis Nelson of UW for pointing out this erratum.
- 47. Page 392, twelfth line from top. Change (4.20) to (4.40).
- 48. Page 393, sixth line from the bottom. Change maximize to maximizing.
- 49. Page 394, tenth line from top. Change  $e_i = y_i - x^T \theta$  to  $e_i = y_i - x_i^T \theta$ .



**Figure 4.10:** The multivariate normal density (top right). The two marginal densities and marginal 95% confidence regions (shaded) (top left and bottom right). The joint elliptical 95% confidence region (shaded), bounding box (outer), and the marginal box (inner) (bottom left).

50. Page 398, Figure 4.10. The joint density in the top right plot is not correct. The corrected figure is shown here. Thanks to Ralph White of Univ. of South Carolina for pointing out this erratum.
51. Page 410, sixth line from top. Change  $B^T B$  to  $BB^T$ .
52. Page 411, twelfth line from top. Change right-hand side from  $R^{-2}C$  to  $R^{-1}C^T R^{-1}$ . Eighth line from bottom, change  $\frac{1}{2}R^{-2} \sum_i (y_i - \Theta x_i)(y_i - \Theta x_i)^T$  to  $\frac{1}{2}R^{-1} \left( \sum_i (y_i - \Theta x_i)(y_i - \Theta x_i)^T \right) R^{-1}$ . Make this same change on page 414, third displayed equation from bottom.
53. Page 411, three lines from bottom. Change “The estimate  $\hat{R}$  is an unbiased estimate of the measurement variance  $R$ .” to “The estimate  $\hat{R}$  is a biased estimate of the measurement variance  $R$  with  $\mathcal{E}(\hat{R}) = R(n - q)/n$ .”
54. Page 412, Equation 4.61. Change  $p_W(W)$  to  $p_W(W, n)$ .
55. Page 414. Third displayed equation from bottom, change  $\frac{1}{2}R^{-2} \sum_i (y_i - \Theta x_i)(y_i - \Theta x_i)^T$  to  $\frac{1}{2}R^{-1} \left( \sum_i (y_i - \Theta x_i)(y_i - \Theta x_i)^T \right) R^{-1}$ .
56. Page 419. In the displayed  $B_{\text{PCR}}$  equation, change the  $P_\ell^T$  to  $P_\ell$ .
57. Page 420. In the two lines preceding the  $B_{\text{PLS}}$  displayed equation, change  $R$  to  $P$  (two places).
58. Page 447, Exercise 4.45. Change  $F(n, p, \alpha)$  to  $F(p, n, \alpha)$  in second displayed equation.
59. Page 447, third line from bottom. Change Exercise 4.34 to Exercise 4.44.

60. Page 458, line before 4th displayed equation. Change  $G = \sqrt{2D}$  to  $G = \sqrt{2D\Delta t}$ . Thanks to Sanjan Gupta of UW for pointing out this erratum.
61. Page 465. In Equation 5.18 and on the line following it, change  $\Delta W(t + \Delta t)$  to  $\Delta W(\Delta t)$ .
62. Page 466,  $df(x(t)) = f(x(t + dt)) - f(x(t))$  should be  $df(x(t)) = f(x(t + dt)) - f(x(t))$ , i.e., move the closing parenthesis.
63. Page 472, Equation 5.40. Change  $\mathcal{A} \cdot \mathbf{x}(t)dt$  to  $\mathcal{A}(\mathbf{x}, t)dt$ .
64. Pages 484 and 485, step 3 on both pages. Add minus signs in front of the corresponding  $(1/r) \ln u$  terms. Thanks to Yuliya Preger of UW for pointing out these errata.
65. Page 487, Equation 5.54. Change the minus sign in front of the  $k_{-1}(c_0 + \varepsilon)$  term to a plus sign. Thanks to Apoorva Sampat of UW for pointing out this erratum.
66. Page 492, change  $r_1(x) = k_1 n_A$  to  $r_1(x) = k_1 n_A n_B$  in first displayed equation. Change  $k_1(n_A + 1)$  to  $k_1(n_A + 1)(n_B + 1)$  and change  $k_1 n_A$  to  $k_1 n_A n_B$  in second displayed equation.
67. Page 493, 7th line. Change  $\xi$  to  $\varepsilon$ .
68. Page 506, first line of second boxed equation. Change  $p_{x(k)|y(k-1)}(x(k+1)|y(k))$  to  $p_{x(k)|y(k-1)}(x(k)|y(k-1))$ .
69. Page 530, Table A.1. Last line on page 530. In the denominator of  $a_{ni}$ , change  $(r_n - 1)!$  to  $(r_n - i)!$ .