

Model Predictive Control: Theory and Design

Errata for First Printing

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November 4, 2017

1. Page 3, five lines from bottom. Add D to $G(s)$ expression: $G(s) = C(sI - A)^{-1}B + D$. Thanks to Jordan Jalving, Ranjeet Kumar, and Apoorva Sampat of UW for pointing out this erratum.
2. Page 16, lines 12, 13, and 16. Put factor $(1/2)$ in front of all terms on the right-hand sides of these equations. Thanks to W. Krämer of T.H. Ingolstadt for pointing out this erratum.
3. Page 18, line 14. Change x_{N-1}^0 to x_N^0 . Page 19, line 7. Change x_{N-2}^0 to x_{N-1}^0 . Thanks to Ganzhou Wang of U. Stuttgart for pointing out this erratum.
4. Page 34, line 8. Remove $C'R^{-1}$ factor in front of $(y(0) - C\bar{x}(0))$. On line 13, remove trailing $C'R^{-1}$ factor.
5. Page 35, first equation. Change first A to A' .
6. Page 36, lines 12 and 13. Change the norm to norm squared on the right-hand sides of the V_k^- and V_k equations.
7. Page 42, second and third displayed equations. Change sign from $-$ to $+$ in front of term $|\mathcal{Y}(T) - C\hat{x}(T|T)|_{R^{-1}}^2$. Thanks to Ganzhou Wang of U. Stuttgart for pointing out this erratum.
8. Page 53, Example 1.11. Change $U\gamma^2$ to $\pi\gamma^2$ in the denominators of the first terms on the right-hand sides of the three differential equations.
9. Page 69, second to last displayed equation. Change $x(k)'P_f x(k)$ term to $x(N)'P_f x(N)$. Thanks to Travis Arnold of UW for pointing out this erratum.
10. Page 76, (1.64). Change P_0 to P_0^{-1} . Thanks to Megan Zagrobelny of UW for pointing out this erratum.
11. Page 77, part (c). Change " $e \sim (0, R)$ " to " $e \sim N(0, R)$ ". Thanks to Cameron Cotten of UW for pointing out this erratum.
12. Page 79, last line. Change $p_{x(0), y(0)}(x(0)|y(0))$ to $p_{x(0)|y(0)}(x(0)|y(0))$. Thanks to Akhil Sundararajan of UW for pointing out this erratum.
13. Page 82, Exercise 1.56. Change z_{sp} to r_{sp} . Make the same change in Exercise 1.57, Lemma 1.14, and Exercise 1.58 (four places: page 83, lines 2, 5, 9, and 10 from the top.). Thanks to W. Krämer of T.H. Ingolstadt for pointing out this erratum.
14. Page 84, Figure 1.13. Change y_t to y'_{sp} .
15. Page 95. In the displayed equations, replace $u^0(N-1; (x, i))$ and $x^0(N; (x, i))$ with $u^0(i+N-1; (x, i))$ and $x^0(i+N; (x, i))$, respectively. Thanks to Phillip Maree of NTNU for pointing out this erratum.
16. Page 97. For added clarity, change Assumption 2.2 to: "The functions $f: \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{R}^n$, $\ell: \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{R}_{\geq 0}$ and $V_f: \mathbb{X}_f \rightarrow \mathbb{R}_{\geq 0}$ are continuous, $f(0, 0) = 0$, $\ell(0, 0) = 0$ and $V_f(0) = 0$." Thanks to Rishi Amrit and Brett Stewart of UW for pointing out this erratum.

17. Page 98, line 3. Change "continuity of $\ell(\cdot)$ " to "continuity of $\ell(\cdot)$ and $V_f(\cdot)$ ". Thanks to He Kong of the University of Newcastle for pointing out this and several of the following errata.
18. Page 102, line 13. Change $x \geq x_{c_2}$ to $x \geq x_{c_2} = 3$.
19. Page 105 and Page 106. Change $(\sin(\theta), \cos(\theta))$ to $(\cos(\theta), \sin(\theta))$ in the captions of Figures 2.3 and 2.4, and the last line of page 106 (three places). Thanks to Cuyler Bates of UW for pointing out this erratum.
20. Page 111, line 10. Change $\mathcal{X}_{N-1} \ni \mathcal{X}_N$ to $\mathcal{X}_{N-1} \subseteq \mathcal{X}_N$.
21. Page 111, line 12. Change (x, u) to $(x, u) \in \mathbb{Z}$.
22. Page 114. Change the sentence after (2.19) to: "A continuous positive definite function $V_f(\cdot)$ satisfying inequality (2.19) for all $x \in \mathbb{R}^n$ with a positive definite $\ell(\cdot)$ is a *global control Lyapunov function* (CLF)."
23. Page 121, sixth line above the first displayed equation. Change " $A^{-1}X \oplus (-A^{-1}B)$ " to " $A^{-1}X \oplus (-A^{-1}BU)$ ". Thanks to Cuyler Bates of UW for pointing out this and several of the following errata.
24. Page 128, last line of the first paragraph in Definition 2.27. Change " $x^+ = f(x, u, i) \in \mathcal{X}(i+1)$." to " $x^+ = f(x, i) \in \mathcal{X}(i+1)$."
25. Page 129, Lemma 2.33. Change $i+N$ to i (two places) in the displayed statement.
26. Page 130, second displayed equation in Assumption 2.34. Change $V_f(x)$ to $V_f(x, i)$.
27. Page 130. Change $V_N^0(f(x, \kappa_N(x, i), i))$ to $V_N^0(f(x, \kappa_N(x, i), i+1))$ in last displayed equation in Proposition 2.35 and (2.32).
28. Page 135, line 8 from the bottom. Change $\mathbb{R}^n \times \mathbb{R}^n$ to $\mathbb{R}^n \times \mathbb{R}^m$.
29. Page 135, line 5 from the bottom. Change V_f to V_N^0 . Thanks to Ganzhou Wang of U. Stuttgart for pointing out this erratum.
30. Page 139, the indented paragraph above the section 2.5.2.2 header. Change " $x^+ = Ax + BK\kappa_N(x)$ " to " $x^+ = Ax + B\kappa_N(x)$ ".
31. Page 140, fourth displayed equation. Change $K := -\kappa B'PA$ to $K := -\kappa PB'A$.
32. Page 144, second displayed equation. Change the sum limit from $j-1$ to $N-1$ and $x(j)$ to $x(N)$.
33. Page 147, eighth line under section 2.6.1. In the statement involving \mathbb{X}_f , change strict subset \subset to subset \subseteq .
34. Page 149, third line. Change "has an interior" to "contains the origin in its interior." Fourth line. Change $\hat{U}_N^c(x)$ to $\hat{U}_N(x)$.
35. Page 149, line after third displayed equation and line after fourth displayed equation. Change $x \in \mathcal{X}_N$ to $x \in \mathcal{X}_N^c$ (two places).
36. Page 151, first displayed equation. In the last inequality, $Nd + \beta$ should be changed to $Nd + \beta a$.
37. Page 151, second to last line of text. Change $\hat{U}_N(x)s$ to $\hat{U}_N(x)$.
38. Page 152, third displayed equation. Change $\kappa_N(x)$ to $\kappa_N^e(x)$.

39. Page 152, second line below the fourth displayed equation. Change " $x^e(N; x^+) = 0 \in \mathcal{X}_N^e$ " so that " $x^+ \in \mathcal{X}_N^e$ " to " $x^e(N; x^+) = 0 \in \mathcal{X}_f$ " so that " $x^+ \in \mathcal{X}_N^e$ ".
40. Page 153, the displayed equation, u and y should be bolded, \mathbf{u} , \mathbf{y} .
41. Page 155, second line below the displayed equation. Add the word "that" after "such".
42. Page 158, first line after displayed equation. Change "or" to "for".
43. Page 160, last line before section 2.9.2. Change " $\mathcal{X}_N := \{x \mid \mathcal{U}_N(x) \neq \emptyset\}$ " to " $\mathcal{X}_N(r) := \{x \mid \mathcal{U}_N(x, r) \neq \emptyset\}$ ".
44. Page 160, last line. Change to $\hat{d}^+ = \hat{d} + L(y - h(x) - \hat{d})$, i.e., move the closing parenthesis. Thanks to Ganzhou Wang of U. Stuttgart for pointing out this and several of the following errata.
45. Page 161, fourth line. Change $+Lv$ to $-Lv$.
46. Page 165. Add following state space realization in addition to $G(s)$ transfer function
- $$A = \begin{bmatrix} 0.857252 & 0.884179 \\ -0.014736 & -0.015139 \end{bmatrix} \quad B = \begin{bmatrix} 8.56490 \\ 0.88418 \end{bmatrix} \quad C = \begin{bmatrix} -0.0043717 & 0 \end{bmatrix}$$
47. Page 168, seventh line. Change "Because quadratic programs ..." to "Because efficient programs exist for solving QPs ...". Thanks to Florian Brunner of U. Stuttgart for pointing out this erratum.
48. Page 172, Exercise 2.4. Change $p = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ to $p = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$.
49. Page 177, Exercise 2.12. Change the text "... the fact that $x^e(N; x)$ lies in \mathcal{X}_N^e " to "... the fact that $x^e(N; x)$ lies in \mathcal{X}_f ".
50. Page 178, Exercise 2.15. Change $N_o \geq 0$ to $N_o \geq 1$.
51. Page 197, second line above third displayed equation. Change "subsets of \mathbb{R}^m " to "subsets of \mathbb{R}^n ".
52. Page 230, 14 lines from bottom. Replace "so that" with "so that, with $|(x, z)| := |x| + |z|$ ". Thanks to Florian Brunner of U. Stuttgart for pointing out this issue. See also Exercises A.47 and A.48 for further discussion of the norm of an extended state.
53. Page 251, second displayed equation. Change term $-kx_1 \exp(-M/x_2)$ to positive sign. Thanks to Larry Ricker of U. Washington for pointing out this erratum.
54. Page 254, Figure 3.8. Change $\Delta = 12$ s and $\Delta = 8$ s to $\Delta = 12$ and $\Delta = 8$, respectively.
55. Page 258, Exercise 3.4. Change second S to $\{y\} \oplus S$.
56. Page 269, Assumption 4.5. Delete "bounded" from the first sentence.
57. Page 274, equation (4.12). Change the final A to A' . Thanks to Travis Arnold of UW for pointing out this erratum.
58. Page 277, second line from bottom. Change $k \leq N$ to $T \leq N$. Thanks to Paola Falugi of Imperial College for pointing out this erratum.
59. Page 279, 7th line from top. Change Exercise 4.11 to Exercise 4.10.
60. Page 279, second displayed equation. Change $y(i) - \hat{x}(i|T)$ to $y(i) - h(\hat{x}(i|T))$. Thanks to Paola Falugi of Imperial College for pointing out this erratum.

61. Page 283. Change Assumption 4.19 to Definition 4.19.
62. Page 287, paragraph before Section 4.3.3. Change "less complex than satisfying the equality (4.16) required in the MHE arrival cost recursion," to "less complex than satisfying the equality in the MHE arrival cost recursion (Definition 4.16)," Thanks to Paola Falugi of Imperial College for pointing out this erratum.
63. Page 292. First displayed equation after Section 4.4. Change $f(\hat{x}(k), u(k))$ to $f(\hat{x}(k), 0)$; second displayed equation; change $\bar{h}(k)$ to $\bar{C}(k)$. Thanks to Luo Ji of UW for pointing out this erratum.
64. Page 293. Second set of displayed equations; change $f(x, u)$ to $f(x, w)$ and add $\bar{G}(k) = \frac{\partial f(x, w)}{\partial w}$. Next line of text; change "and $u(k)$, and $\bar{h}(k) = h(\hat{x}(k), u(k))$." to "and $w = 0$."
65. Page 294. Second to last displayed equation; replace with $z^i(k+1) = f(z^i(k), n^i(k))$.
66. Page 295. Fourth line after last displayed equation. Replace $\partial f(x, u)/\partial x$ with $\partial f(x, w)/\partial x$.
67. Pages 308, 309. Change $\binom{i}{s}$ to $\binom{s}{i}$ in (4.40) and in the second sentence on page 309 (two places). Thanks to Megan Zagrobelny of UW for pointing out this erratum.
68. Pages 319, 323. Insert missing factor of $(1/2)$ in the exponential terms in the following equations: 4th line from bottom and last line of page 319, first and second displayed equation on page 323. Thanks to Ming Ge of Imperial College for pointing out this erratum.
69. Page 328, line 16. Change Exercise 4.33 to Example 4.33. Thanks to Ming Ge of Imperial College for pointing out this erratum.
70. Page 331, line 3 in boxed recursion. It's numerically better to store and use the normalized weights in the recursion. Therefore change $w_i(k) = w_i(k-1)p(y(k)|x_i(k))$ to $w_i(k) = \bar{w}_i(k-1)p(y(k)|x_i(k))$. See also page 343.
71. Page 334, last line. Change $dx_i(k-1)$ to $dx_i(k)$. Thanks to Ming Ge of Imperial College for pointing out this erratum.
72. Page 337, fourth line. Change final Q to GQG' . Thanks to Ming Ge of Imperial College for pointing out this erratum.
73. Page 340. Starting at line 7, change

in which we have used the identity (see Exercise 1.47)

$$p_{A|B}(a|b) = \int p_{A|B,C}(a|b, c) p_{B|C}(b|c) dc$$

to

in which we have used the second identity in Exercise 1.47 and the Markov property, which implies that

$$p(y(k+1)|\mathbf{x}(k+1), y(k)) = p(y(k+1)|\mathbf{x}(k+1))$$

Thanks to Ming Ge of Imperial College for pointing out this erratum.

74. Page 343, line 4 in boxed recursion. Change $w_i(k+1) = w_i(k) \frac{p(y(k+1)|x_i(k+1))p(x_i(k+1)|x_i(k))}{q(x_i(k+1)|x_i(k), y(k+1))}$ to $w_i(k+1) = \bar{w}_i(k) \frac{p(y(k+1)|x_i(k+1))p(x_i(k+1)|x_i(k))}{q(x_i(k+1)|x_i(k), y(k+1))}$. Thanks to Ming Ge of Imperial College for pointing out this issue. See also page 331.

75. Page 343, last sentence before 4.7.6. Change “increases with sample size” to “increases with time”.
76. Page 346, second displayed equation. Change P to \bar{P} . Thanks to Ming Ge of Imperial College for pointing out this erratum.
77. Page 364, Exercise 4.25, third displayed equation. Change P to \bar{P} (twice). Thanks to Ming Ge of Imperial College for pointing out this erratum.
78. Page 373, last line. Change \bar{x} to \hat{x} . Thanks to Paola Falugi of Imperial College for pointing out this erratum and several of the Chapter 5 errata that follow.
79. Page 380, line 6 of paragraph after Proposition 5.3. Change $\tilde{x}(0) = \hat{x}(0) - z(0)$ to $\tilde{x}(0) = x(0) - \hat{x}(0)$.
80. Page 381, second line from bottom. Replace $V_N^0(\cdot)$ with $\bar{V}_N^0(\cdot)$, and $\kappa_N(\cdot)$ with $\bar{\kappa}_N(\cdot)$. Last line. Replace $\mathcal{V}_N^0(z)$ with $\mathcal{V}_N(z)$.
81. Page 382, Step 1 of robust control algorithm. Change $K(x - z)$ to $K(\hat{x} - z)$. Thanks to He Kong of the University of Newcastle for pointing out this erratum.
82. Page 384, first line after last displayed equation. Change $\phi(0) \in \mathbb{F}$ to $\phi(0) \in \Phi$.
83. Page 386 (middle) and Page 395 (top). Change $A - BK$ to $A + BK$.
84. Page 386, second line from bottom. Change “ $\tilde{e}(0)$ lies $\Sigma(0)$ ” to “ $\tilde{x}(0)$ lies in $\Sigma(0)$ ”.
85. Page 387, third line. Change $\phi(i) = (\tilde{x}(0), e(0))$ to $\phi(i) = (\tilde{x}(i), e(i))$.
86. Page 389, line before third displayed equation. Change $\kappa_N(\cdot, k)$ to $\bar{\kappa}_N(\cdot, k)$. In third displayed equation, change $\mathcal{V}_N^0(z, k)$ to $\mathcal{V}_N(z, k)$.
87. Page 389, lines 4, 5 and 6 from the bottom. Change ΔV_N^0 and V_N^0 to $\Delta \bar{V}_N^0$ and \bar{V}_N^0 , respectively.
88. Page 390, third line before Section 5.4.5. Change $\kappa_N(z, k)$ to $\bar{\kappa}_N(z, k)$.
89. Page 393, line 16. Change $\tilde{y} = \tilde{C}\phi + \nu$ to $\tilde{y} = \tilde{C}\tilde{\phi} + \nu$.
90. Page 393, line 11 from the bottom. Change $\tilde{w} = L(\tilde{C}\tilde{\phi} + \nu)$ to $\tilde{w} = w - Lv$.
91. Page 393, lines 7 and 8 from the bottom. Change Δ to \tilde{w} (two places) and δ to \tilde{w} (two places).
92. Page 394, \hat{x}^+ displayed equation. Change $A\hat{x}$ to $A\hat{x} + B_d\hat{d}$. Page 398, line 16, change $A\hat{x}$ to $A\hat{x} + B_d\hat{d}$. Thanks to John Jørgensen of the Danish Technical University for pointing out this erratum.
93. Page 394, z^+ displayed equation. Change Az to $Az + B_d\hat{d}$. Page 396, line 2, Page 398, lines 1, 9, 13, and 16, change Az to $Az + B_d\hat{d}$.
Bottom of Page 395, change $A\bar{z}$ to $A\bar{z} + B_d\hat{d}$.
94. Page 395, line 19. Change Σ_d to $i\Sigma_d$.
95. Page 395, line 2 of **Target Calculation**. Change $\bar{\kappa}_N(\hat{x}, z)$ to $\bar{\kappa}_N(z, k)$.
96. Page 396, line 7 of **MPC algorithm**. Change $V_N(z, \hat{d}, \mathbf{v})$ to $V_N(z, \hat{d}, \bar{r}, \mathbf{v})$.
97. Page 396, line 8 of **MPC algorithm**. Change $\mathcal{V}(z, \hat{d})$ to $\mathcal{V}(z, \hat{d}, \bar{r})$.
98. Page 396. Change this sentence

- where, for each i , $z(i) = \tilde{\phi}(i; z, \mathbf{v})$, the solution of $z^+ = Az + Bv$ when the initial state is z and the control sequence is \mathbf{v} .
- to
where, for each i , $z(i) = \tilde{\phi}(i; z, \hat{d}, \mathbf{v})$, the solution of $z^+ = Az + B_d\hat{d} + Bv$ when the initial state is z , the control sequence is \mathbf{v} , and the disturbance \hat{d} is constant.
99. Page 396, line 8 from bottom. Change \mathbb{P}_N to $\bar{\mathbb{P}}_N$. Page 397, line 1. Same change.
100. Page 396, line 3 from bottom. Change $\kappa_N(\hat{x}, z, \hat{d})$ to $\kappa_N(\hat{x}, z, \hat{d}, \bar{r})$.
101. Page 397, Step 4. Change $z^+ = f(z, v)$ to $z^+ = Az + B_d\hat{d} + Bv$.
102. Page 398, line 1. Change $\bar{\kappa}_N(z, \hat{d})$ to $\bar{\kappa}_N(z, \hat{d}, \bar{r})$.
103. Page 398, line 4. Change $\bar{\mathbb{P}}_N(z, \hat{d})$ to $\bar{\mathbb{P}}_N(z, \hat{d}, \bar{r})$.
104. Page 398, last displayed equation. Change

$$\Delta V_N^0(z, \hat{d}, \bar{r}, w_d) = V_N^0(Az + B_d\hat{d} + B\bar{\kappa}_N(z, \hat{d}, \bar{r}), \hat{d} + w_d) - V_N^0(z, \hat{d}, \bar{r})$$

$$\leq -c_1 \left| z - \bar{z}(\hat{d}, \bar{r}) \right|^2 + \bar{k} |w_d|$$
where \bar{k} is a Lipschitz constant for $V_N^0(\cdot, \hat{d}, \bar{r})$ for all $(\hat{d}, \bar{r}) \in X_d \times X_r$, and X_r is the set of permissible values for \bar{r} .
- to

$$\Delta V_N^0(z, \hat{d}, \bar{r}, w_d) = V_N^0(Az + B_d\hat{d} + B\bar{\kappa}_N(z, \hat{d}, \bar{r}), \hat{d} + \delta_d, \bar{r}) - V_N^0(z, \hat{d}, \bar{r})$$

$$\leq -c_1 \left| z - \bar{z}(\hat{d}, \bar{r}) \right|^2 + \bar{k} |\delta_d|$$
where \bar{k} is a Lipschitz constant for $V_N^0(\cdot)$.
105. Page 398, last paragraph. Change \mathbb{W}_d to Δ_d (two places).
106. Page 437, last two lines of last displayed equation. Include missing factors $(1/2)\rho_1$ and $(1/2)\rho_2$ in front of $x_1(N)$ and $x_2(N)$ terms. Thanks to Ganzhou Wang for pointing out this erratum.
107. Page 447, Assumption 6.12. Change (a) to: The systems $(\underline{A}_i, \underline{B}_i)$, $i = 1, 2$ are stabilizable, in which $\underline{A}_i = \text{diag}(A_{1i}, A_{2i})$ and $\underline{B}_i = \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix}$. Change (e) to: $N \geq \max_{i \in \{1, 2\}} \underline{n}_i^u$, in which \underline{n}_i^u is the number of unstable modes of \underline{A}_i .
108. Page 447. Delete the following sentence.
Let Σ_{ij} denote the solution of the Lyapunov equation

$$A_{ij}^s \Sigma_{ij} A_{ij}^s - \Sigma_{ij} = -S_{ij}^s Q_{ij} S_{ij}^s$$
Then change the first two sentences on Page 448 from:
We further define the matrices

$$\Sigma_i = \text{diag}(\Sigma_{i1}, \Sigma_{i2}) \quad i \in \{1, 2\}$$
These matrices satisfy the Lyapunov equations
to the single sentence:

We further define the matrices Σ_1, Σ_2 as the solutions to the Lyapunov equations

Thanks to Brett Stewart of UW for pointing out this erratum.

109. Page 448. Change $u_1 \in \mathcal{U}_1$ $u_2 \in \mathcal{U}_2$ to $\mathbf{u}_1 \in \mathcal{U}_1$ $\mathbf{u}_2 \in \mathcal{U}_2$ in the displayed equation before the last paragraph.
110. Page 450, last two lines of last displayed equation. Include missing factors ρ_1 and ρ_2 in front of $x_1(N)$ and $x_2(N)$ terms. Same change on first displayed equation on Page 451. Thanks to Ganzhou Wang for pointing out this erratum.
111. Page 452, Figure 6.6. Change Ω to \mathbb{U} .
112. Page 457, optimization problem (6.28). Change min argument x_{1s} to x_{11s}, x_{21s} .
113. Page 460, Assumption 6.16. Change (a) to: The systems $(\underline{A}_i, \underline{B}_i)$, $i \in \mathbb{1}_{1:M}$ are stabilizable, in which $\underline{A}_i = \text{diag}(A_{1i}, A_{2i}, \dots, A_{Mi})$. Change (b) to: (A_i, C_i) , $i \in \mathbb{1}_{1:M}$ are detectable. Change (e) to: $N \geq \max_{i \in \mathbb{1}_{1:M}} (\underline{n}_i^u)$, in which \underline{n}_i^u is the number of unstable modes of \underline{A}_i .
114. Page 477, Figure 6.9. Change Ω to \mathbb{U} .
115. Page 479, Exercise 6.30. Change argument x_{1s} to x_{11s}, x_{21s} . Delete the hint.
116. Page 495. Line after (7.5). Change P_x^0 to p_x^0 .
117. Page 499, Proposition 7.13. Change the last sentence to "Moreover, the value function and the minimizer are Lipschitz continuous on bounded sets."
118. Page 517. Exercise 7.7 (a). Change entry in perturbed D matrix from $-(1 + \epsilon)$ to $-1 + \epsilon$. Thanks to Brett Stewart of UW for pointing out this erratum.
119. Page 625. Exercise B.4. Add statement that $f(\cdot)$ is continuous.
120. Page 629. Missing 16 on leg on far right side, 3rd from bottom. Thanks to Travis Arnold of UW for pointing out this erratum.