In search of the unreachable setpoint

Adventures with Prof. Sten Bay Jørgensen

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Seminar Honoring Prof. Sten Bay Jørgensen
CAPEC
Technical University of Denmark

Outline

The first paper

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 53, NO. 9, OCTOBER 2008

Unreachable Setpoints in Model Predictive Control

James B. Rawlings, Dennis Bonné, John B. Jørgensen,
Aswin N. Venkat, and Sten Bay Jørgensen

***Abstract*—In this work, a new model predictive controller is developed that handles unreachable setpoints better than traditional model predictive control methods. The new controller induces an interesting fast/slow asymmetry in the tracking response of the system. Nominal asymptotic stability of the optimal steady state is established for terminal constraint model predictive control (MPC). The region of attraction is the steerable set. Existing analysis methods for closed-loop properties of MPC are not applicable to this new formulation, and a new analysis method is developed. It is shown how to extend this analysis to terminal penalty MPC. Two examples are presented that demonstrate the advantages of the proposed setpoint-tracking MPC over the current target-tracking MPC.**

***Index Terms*—Aymptotic stability, constraints, model predictive control.**

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- 8 1/2 years since John first arrived in Madison

Setpoints and unreachable setpoints

Setpoints and unreachable setpoints

Consider the steady state of a dynamic model with state x, controlled input u, and disturbance w

$$x(k+1) = Ax(k) + Bu(k) + B_d w(k)$$

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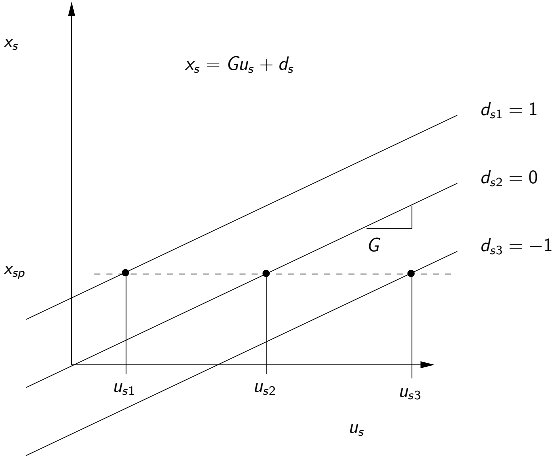
$$x(k+1) = Ax(k) + Bu(k) + B_d w(k)$$

$$x_s = \underbrace{(I-A)^{-1}B}_{G}\, u_s + \underbrace{(I-A)^{-1}B_d w_s}_{d_s}$$

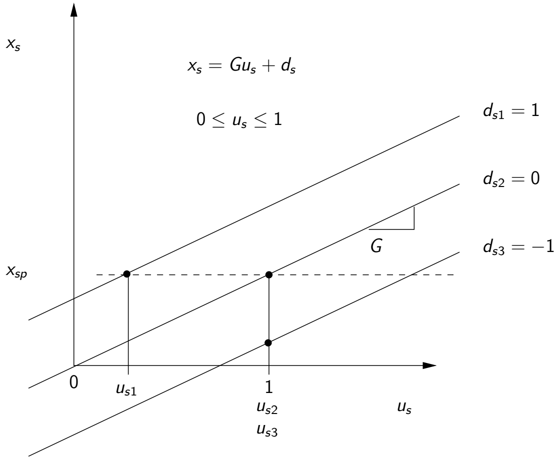
$$x_s = Gu_s + d_s$$

Steady states — unconstrained system



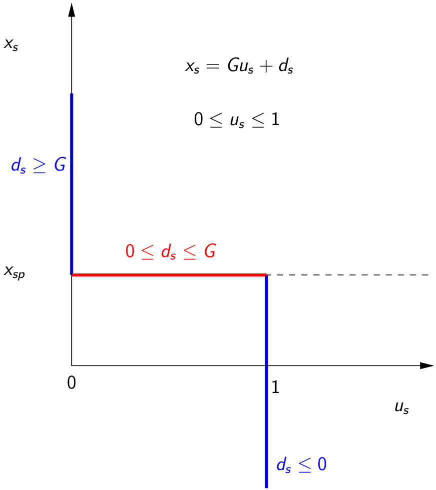
For an unconstrained system with $G \neq 0$, any setpoint x_{sp} with any disturbance d_s has a corresponding u_s.

Constraints and unreachable setpoints



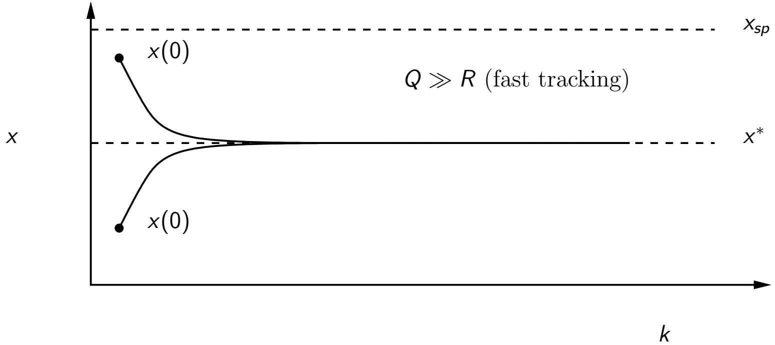
For a *constrained* system, the setpoint x_{sp} may be unreachable for a given disturbance d_s. MPC is method of choice for this situation.

Constraints and unreachable setpoints

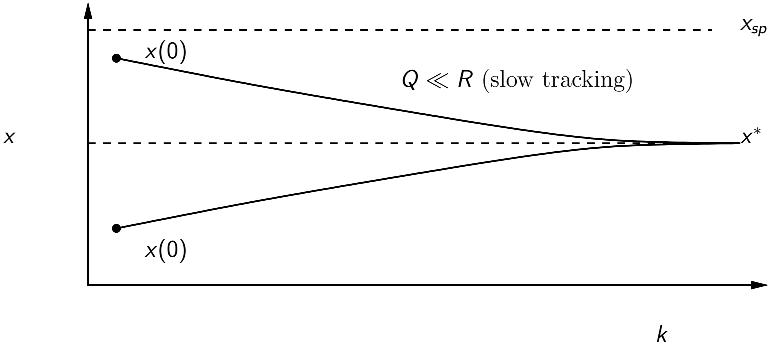


As the estimated disturbance changes with time, the setpoint may change between reachable and unreachable.

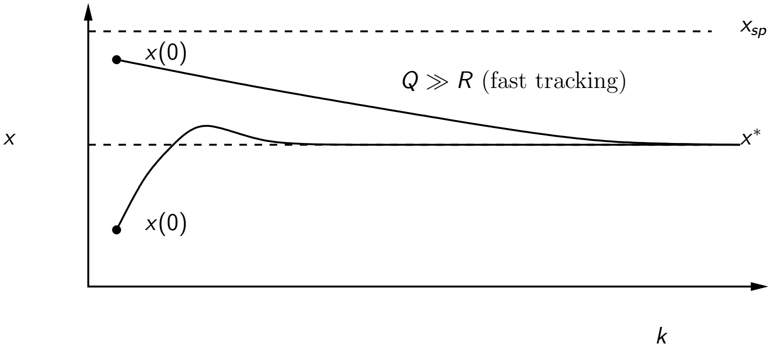
What closed-loop behavior is desirable? Fast tracking



What closed-loop behavior is desirable? Slow tracking



What closed-loop behavior is desirable? Asymmetric tracking



Why analysis? Unexpected closed-loop behavior

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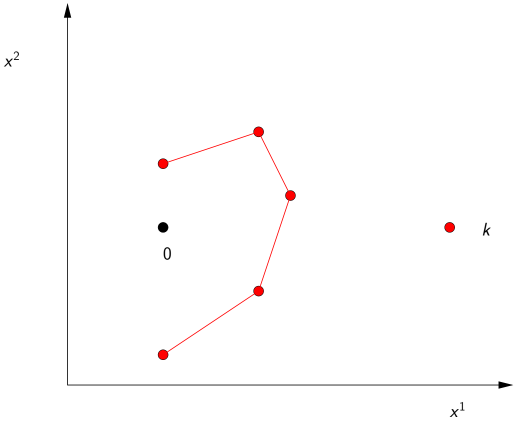

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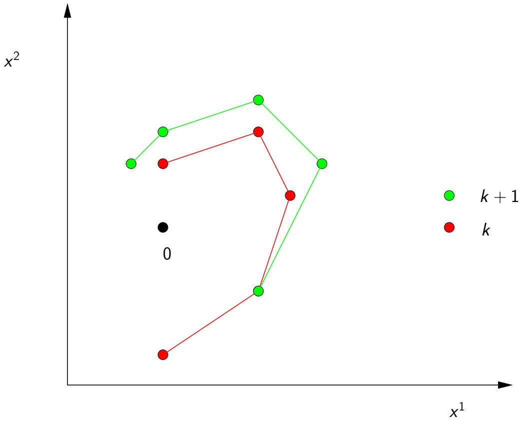

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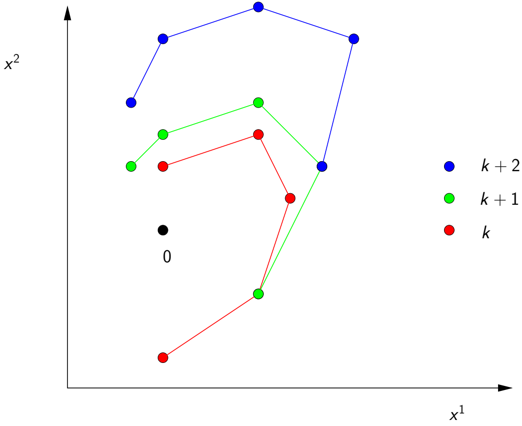

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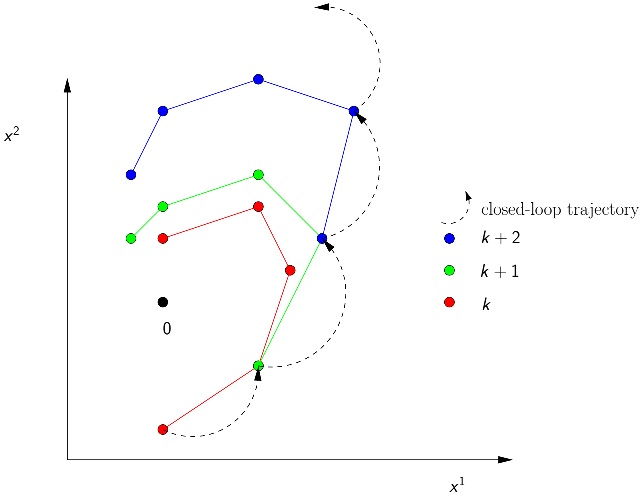

Unreachable case — challenges for analyzing closed-loop behavior

- Sequence of optimal costs is not monotone decreasing

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- Standard nominal MPC stability arguments do not apply
- Simulations indicate the closed loop *is* stable
- How can we be sure?

Unreachable case — theoretical result

Theorem (Asymptotic Stability of Terminal Constraint MPC)

The optimal steady state is the asymptotically stable solution of the closed-loop system under terminal constraint MPC. Its region of attraction is the steerable set.

Some funny emails during the long journey

```
From: "James B. Rawlings" <jbraw@bahaha.che.wisc.edu>
To: John Bagterp <bagterp@bevo.che.wisc.edu>
Date: Wed, 2 Aug 2000 07:10:34 -0500 (CDT)
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From: "James B. Rawlings" <jbraw@bahaha.che.wisc.edu>
To: John Bagterp <bagterp@bevo.che.wisc.edu>
Date: Wed, 2 Aug 2000 07:10:34 -0500 (CDT)

p.s. If Sten doesn't get excited about the approach
pretty soon, you should check his food;
maybe he's on some medication.
```

"Let no one ignorant of geometry enter"

– Engraved over the door of Plato's Academy

Dennis takes a crack

`On  5-Aug-2002, Dennis Bonne <db@olivia.kt.dtu.dk> wrote:`

Dennis takes a crack

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"No one ignorant of geometry shall" ... come to Madison.
```

The end is finally in sight

```
Subject: ieee article proofs
From: "James B. Rawlings" <rawlings@engr.wisc.edu>
To: John Bagterp Jorgensen <jbj@imm.dtu.dk>,
    Sten Bay Jorgensen <sbj@kt.dtu.dk>,
    Dennis Bonn <dennis_bonne@yahoo.dk>,
    Aswin.Venkat@shell.com
Date: Tue, 16 Sep 2008 11:45:00 -0500
```

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Date: Tue, 16 Sep 2008 11:45:00 -0500

Guys,

Gentle reminder: we don't want to let the grass grow
under our feet on this one.

Jim
```

All's well that ends well

```
Subject: Re: forwarded message from jsinay@ieee.org
From: sbj@kt.dtu.dk
To: "James B. Rawlings" <rawlings@engr.wisc.edu>
Date: Thu, 18 Sep 2008 21:34:14 +0200 (CEST)

Dear Jim,
I do hope the mower is not out yet!
```

All's well that ends well

Subject: Re: forwarded message from jsinay@ieee.org
From: sbj@kt.dtu.dk
To: "James B. Rawlings" <rawlings@engr.wisc.edu>
Date: Thu, 18 Sep 2008 21:34:14 +0200 (CEST)

Dear Jim,
I do hope the mower is not out yet!

This is indeed a fine paper!
Thanks for you stamina in making this come through!

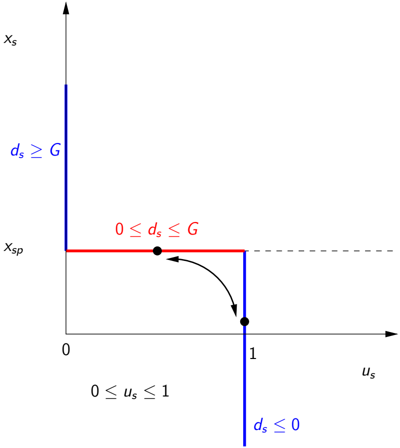
Sincerely
Sten

Example 1. Single input–single output system

$$G(s) = \frac{-0.2623}{60s^2 + 59.2s + 1}$$

- Sample time $T = 10$ sec
- Input constraint, $-1 \leq u \leq 1$
- Setpoint $y_{\text{sp}} = 0.25$
- $Q_y = 10, R = 0, S = 1, Q = C' Q_y C + 0.01 I_2$
- Horizon length $N = 80$
- Periodic state disturbance $d_x = [17.1 \;\; 1.77]'$ which is estimated from the measurements

As the estimated disturbance changes with time, the setpoint changes between reachable and unreachable.



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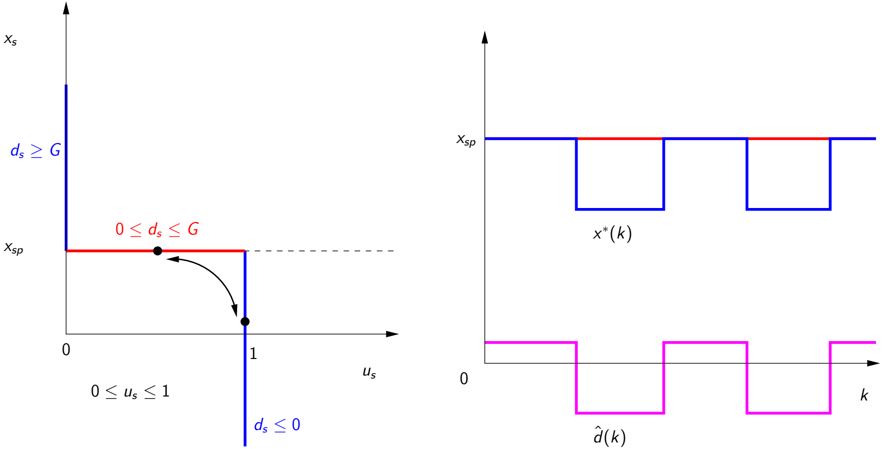
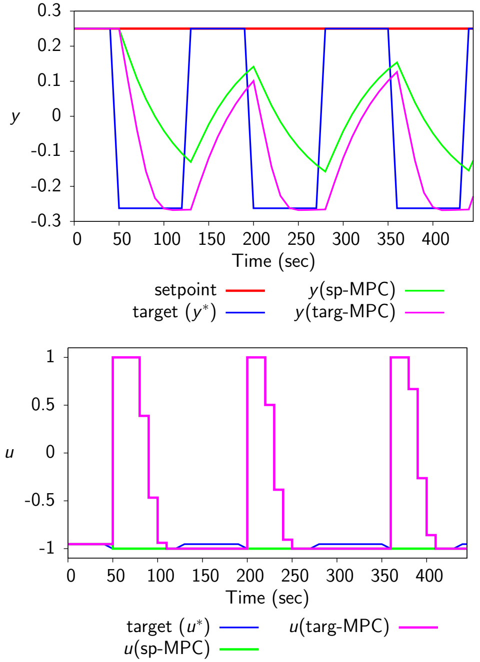
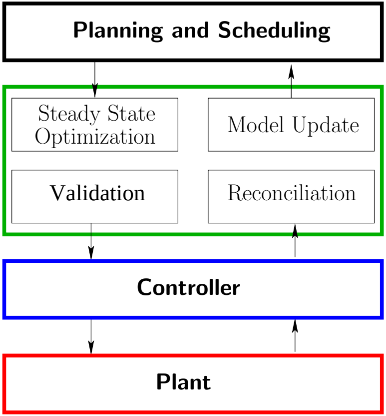



0.3
0.2
0.1
y
0
-0.1
-0.2
-0.3
0
50
100
150
200
250
300
350
400
Time (sec)
setpoint
target (y^*)
y(sp-MPC)
y(targ-MPC)
1
0.5
u
0
-0.5
-1
0
50
100
150
200
250
300
350
400
Time (sec)
target (u^*)
u(sp-MPC)
u(targ-MPC)

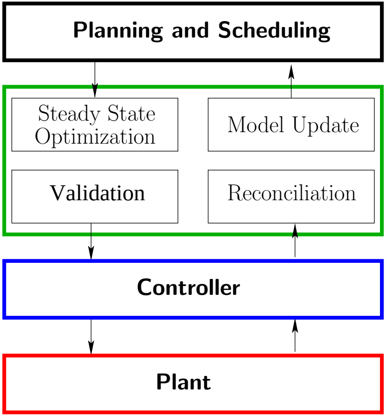
Summary of Example 1

Performance Measure	targ-MPC	sp-MPC	Δ(index)%
V_u	0.016	2.2×10^{-6}	99.98
V_y	3.65	1.71	53
V	3.67	1.71	54

Optimizing economics: Current industrial practice

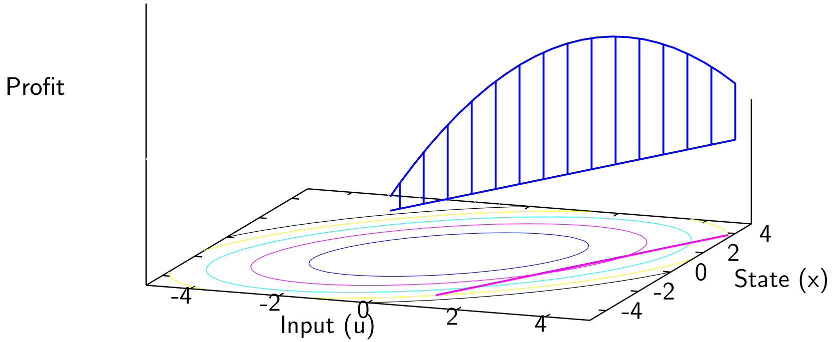


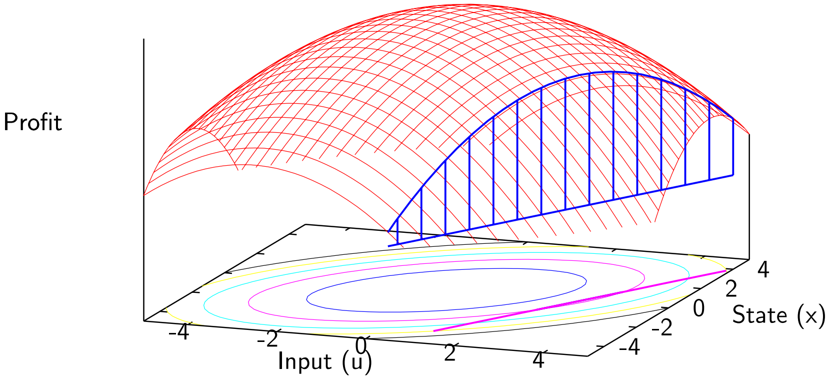
- Two layer structure
- Drawbacks



- Two layer structure
- Drawbacks
  - Inconsistent models
  - Re-identify linear model as setpoint changes
  - Time scale separation may not hold
  - Economics unavailable in dynamic layer

An economics controller




Profit
-4
-2
0
2
4
Input (u)
4
2
0
-2
-4
State (x)

The second paper

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Parameter estimation in stochastic grey-box models☆

Niels Rode Kristensen[a], Henrik Madsen[b,*], Sten Bay Jørgensen[a]

[a] *Department of Chemical Engineering, Technical University of Denmark, Building 229, DK-2800 Lyngby, Denmark*

[b] *Informatics and Mathematical Modelling, Technical University of Denmark, Building 321, DK-2800 Lyngby, Denmark*

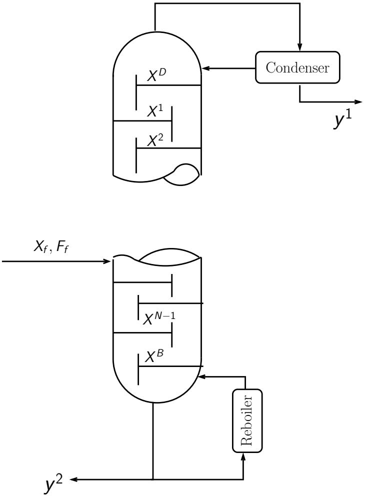
Received 20 November 2002; received in revised form 28 April 2003; accepted 25 September 2003

Abstract

An efficient and flexible parameter estimation scheme for grey-box models in the sense of discretely, partially observed Itô stochastic differential equations with measurement noise is presented along with a corresponding software implementation. The estimation scheme is based on the extended Kalman filter and features *maximum likelihood* as well as *maximum a posteriori* estimation on multiple independent data sets, including irregularly sampled data sets and data sets with occasional outliers and missing observations. The software implementation is compared to an existing software tool and proves to have better performance both in terms of quality of estimates for nonlinear systems with significant diffusion and in terms of reproducibility. In particular, the new tool provides more accurate and more consistent estimates of the parameters of the diffusion term.

Keywords: Grey-box models; Parameter estimation; Stochastic differential equations; Maximum likelihood estimation; Extended Kalman filter; Estimation with missing observations; Robust estimation; Estimation accuracy; Software tools

Obtaining Q and R from Data



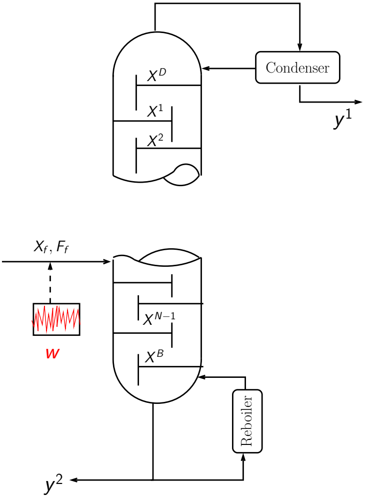
- Model discretized with $t_k = k\Delta t$:

$$\frac{d}{dt}\underbrace{\begin{bmatrix} X^D \\ \vdots \\ X^B \end{bmatrix}}_{x(t)} = F(x(t), \underbrace{u(t)}_{X_f, F_f})$$

$$\begin{bmatrix} y^1 \\ y^2 \end{bmatrix}(t_k) = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} x(t_k)$$

- Measurements are only X^D, X^B at the discretization times

Obtaining Q and R from Data



- Model discretized with $t_k = k\Delta t$:

$$x_{k+1} = f(x_k, u_k) + g(x_k, u_k)w_k$$
$$\begin{bmatrix} y^1 \\ y^2 \end{bmatrix}_k = \begin{bmatrix} 1 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} x_k$$

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- Noise w_k affects all the states

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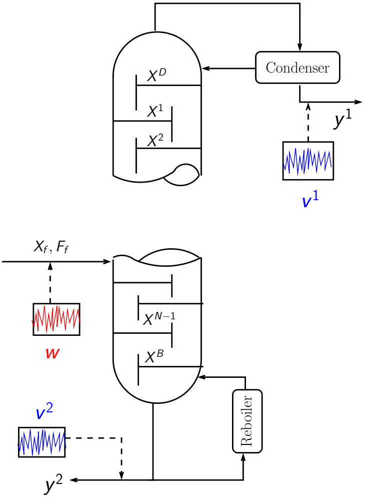

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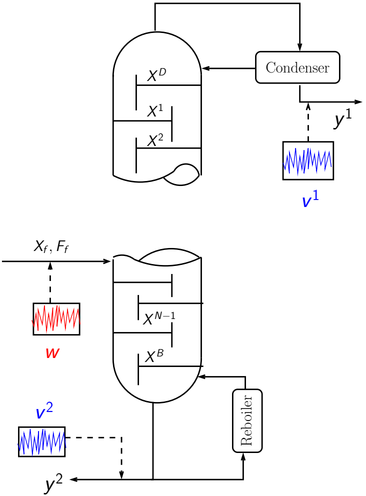
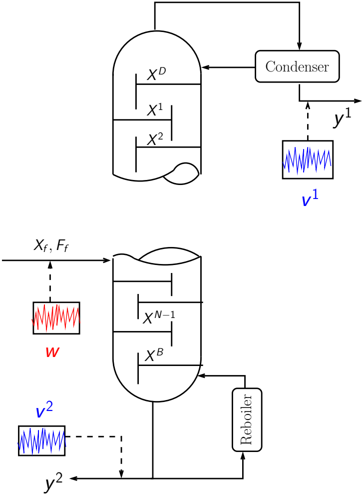
- Noise w_k affects all the states
- Noise v_k corrupts the measurements

Motivation for Using Autocovariances



Idea of Autocovariances

- The state noise w_k gets propagated in time
- The measurement noise v_k appears only at the sampling times and is not propagated in time
- Taking autocovariances of data at different time lags gives covariances of w_k and v_k



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Let w_k, v_k have zero means and covariances Q and R

Mathematical Formulation of the ALS

Linear State-Space Model:

$$x_{k+1} = Ax_k + Gw_k \qquad w_k \sim N(0, Q)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R)$$

- Model (A, C, G) known from the linearization, finite set of measurements: $\{y_0, \dots, y_k\}$ given.
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- $y_{k+1} = CAx_k + CG w_k$
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- $E[y_{k+2} y_{k+1}^T] = CAGQG^T C^T$

The Autocovariance Least-Squares (ALS) Problem

Skipping a lot of algebra, we can write:

Autocovariance Least Squares

$$\Phi = \min_{Q,R} \left\| \mathscr{A}_N \begin{bmatrix} (Q)_s \\ (R)_s \end{bmatrix} - \hat{b} \right\|^2$$

1. A least-squares problem in a vector of unknowns, Q, R

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1. A least-squares problem in a vector of unknowns, Q, R
2. Form $\mathscr{A}_N$ from known system matrices

The Autocovariance Least-Squares (ALS) Problem

Skipping a lot of algebra, we can write:

Autocovariance Least Squares

$$\Phi = \min_{Q,R} \left\| \mathscr{A}_N \begin{bmatrix} (Q)_s \\ (R)_s \end{bmatrix} - \hat{b} \right\|^2$$

1. A least-squares problem in a vector of unknowns, Q, R
2. Form $\mathscr{A}_N$ from known system matrices
3. $\hat{b}$ is a vector containing the estimated correlations from data

$$\hat{b} = \frac{1}{T} \sum_{k=1}^{T} \begin{bmatrix} y_k y_k^T \\ \vdots \\ y_{k+N-1} y_k^T \end{bmatrix}_s$$

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- Preliminary results from Eastman chemical cracking furnace and Aspentech evaluation are encouraging

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- Choosing the right problems is the most important task in research
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- The research climate at DTU and CAPEC is first rate