

# Optimizing Process Economic Performance with Model Based Control

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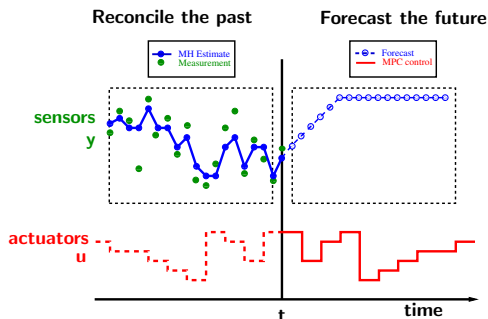


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Model Based Control Conference  
Danish Technical University

- 1 Optimal control, optimal feedback control, and model predictive control (MPC)
- 2 Industrial impact of these ideas
- 3 Using MPC to optimize plant economics
- 4 Summary Remarks

# Predictive control



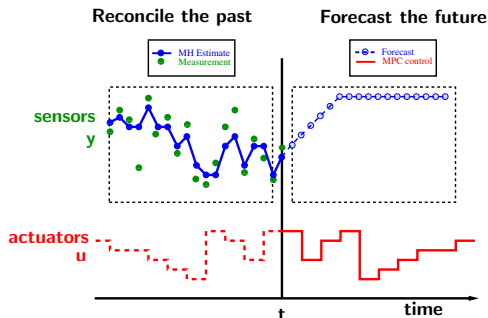
$$\min_{u(t)} \int_0^T |y_{sp} - g(x, u)|_Q^2 + |u_{sp} - u|_R^2 dt$$

$$\dot{x} = f(x, u)$$

$$x(0) = x_0 \quad (\text{given})$$

$$y = g(x, u)$$

# State estimation



$$\min_{x_0, w(t)} \int_{-T}^0 |y - g(x, u)|_R^2 + |\dot{x} - f(x, u)|_Q^2 dt$$

$$\dot{x} = f(x, u) + w \quad (\text{process noise})$$

$$y = g(x, u) + v \quad (\text{measurement noise})$$

## From optimal control to optimal *feedback* control

- Optimal control is developed in the 1960s: adjoint equations, Hamiltonian, maximum principle, etc. Treats nonlinear and constrained systems. (Bryson and Ho, 1975)
- Optimal *feedback* control for *linear, unconstrained* systems also developed in the 1960s: linear quadratic regulator (Kalman, 1960; Kwakernaak and Sivan, 1972)

$$u^0 = Kx$$

- Industrial systems are either constrained or nonlinear or both. Optimal *feedback* control for these systems seems to lead to *intractable* dynamic programming problems. The curse of dimensionality. (Bellman and Dreyfus, 1962)
- Optimal feedback control sees limited industrial application during this period.

## So what unchained optimal control?

*One technique for obtaining a feedback controller synthesis from knowledge of open-loop controllers is to measure the current control process state and then compute **very rapidly** for the open-loop control function. The first portion of this function is then used during a short time interval, after which a new measurement of the process state is made and a new open-loop control function is computed for this new measurement. The procedure is then repeated.*

— Lee and Markus (1967)  
*Foundations of Optimal Control Theory*

Our notion of **very rapidly** changed radically from 1960 to 1985.

# Large industrial success story!

## Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M\$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

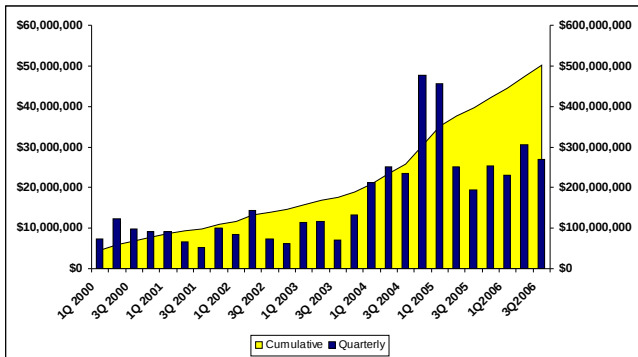
## Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity
- 30-50 M\$/year increased profit due to increased throughput (2008)

## Praxair experience with MPC

- Praxair currently has more than 150 MPC installations
- 16 M\$/year increased profit (2008)

## We're Doing it For the Money

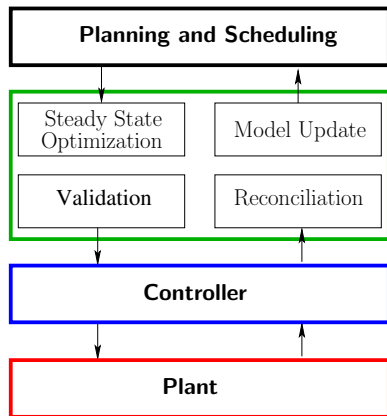


# Are all the problems solved?

## Some questions to consider

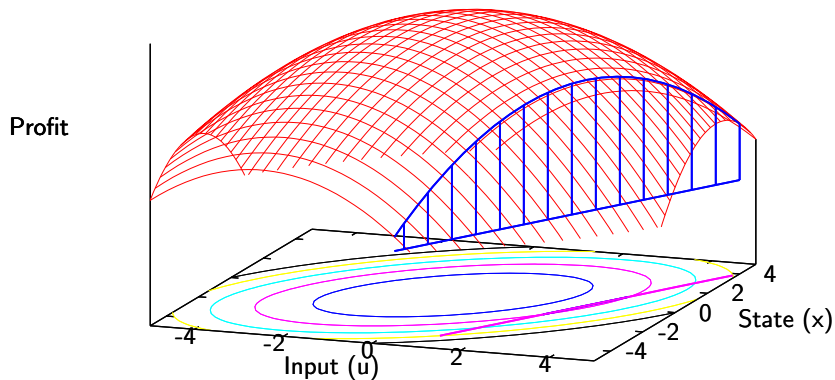
- Has the application base stopped growing?
- Is the theory complete?
- Do we have tools to solve *nonconvex optimization problems* online?
- Do we have tools to decompose *large-scale systems* into manageable problems?
- Do we have tools to *commission* and *maintain* the controllers?
- Do we have tools to optimize dynamic *economic* operation?

# Optimizing economics: Current industrial practice



- Two layer structure
- Drawbacks
  - ▶ Inconsistent models
  - ▶ Re-identify linear model as setpoint changes
  - ▶ Time scale separation may not hold
  - ▶ Economics unavailable in dynamic layer

# Optimizing economics: what's desirable?



# Steady-state problem definition

- Stage cost:

$$\ell(x, u) = \text{plant profit function}$$

- Optimization:  $\min_{x, u} \ell(x, u)$   
subject to:  $x = f(x, u) \quad x \in \mathbb{X} \quad u \in \mathbb{U}$
- Solution:  $(x_s, u_s)$

## MPC problem definition — dynamic case

- Cost function: 
$$V = \sum_{j=0}^{N-1} \tilde{\ell}(x(j), u(j))$$

- Optimization: 
$$\min_{\mathbf{u}} V(\mathbf{u}, x(0))$$

subject to:

$$\begin{aligned} \mathbf{u} &= \{u(0), u(1), \dots, u(N-1)\} \\ x^+ &= f(x, u) \quad \mathbf{x} \in \mathbb{X}^N \quad \mathbf{u} \in \mathbb{U}^N \end{aligned}$$

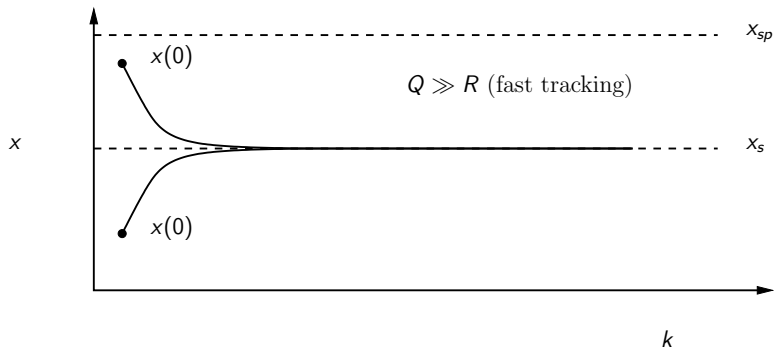
- Stage cost:

$$\text{std-MPC: } \tilde{\ell}(x, u) = |x - x_s|_Q^2 + |u - u_s|_R^2 \quad \text{—or—}$$

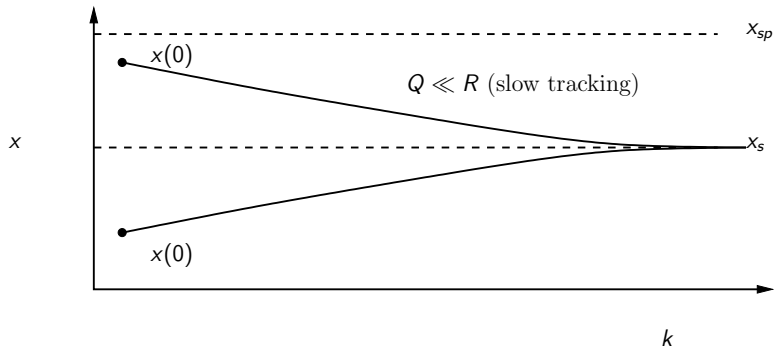
$$\text{eco-MPC: } \tilde{\ell}(x, u) = \ell(x, u)$$

- Control law:  $u^0(x) = \mathbf{u}^0(0; x)$

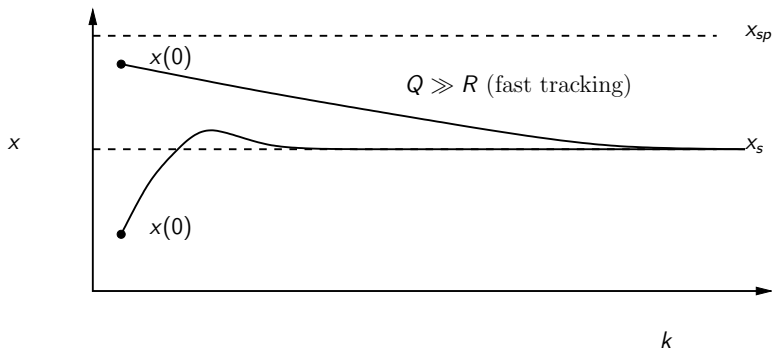
# What closed-loop behavior is desirable? Fast tracking



# What closed-loop behavior is desirable? Slow tracking



# What closed-loop behavior is desirable? Asymmetric tracking



## Unreachable case — challenges for analyzing closed-loop behavior

- Sequence of optimal costs is not monotone decreasing
- Infinite horizon cost is unbounded for all input sequences
- Optimal cost is not a Lyapunov function for the closed-loop system
- Standard nominal MPC stability arguments do not apply
- Simulations indicate the closed loop *is* stable
- How can we be sure?

## Recent results for economic MPC

- The economic MPC for **linear dynamics**,  $f(x, u) = Ax + Bu$ , and **convex cost** is asymptotically stabilizing (Rawlings, Bonn e, J orgensen, Venkat, and J orgensen, 2008)
- Proof based on convexity of stage cost
- No Lyapunov function was found

## A brief history of this paper

- John Jørgensen visits Madison. January–August 2000
- Dennis Bonné visits Madison. February–August 2002
- Paper is submitted to IEEE TAC. July 2007
- Paper is submitted again. November 2007
- First reviews come back. February 2008
- Revised paper submitted. February 2008
- Paper is accepted. April 2008
- Paper appears. October 2008
- 8 1/2 years since John first arrived in Madison

## Some funny emails during this long journey

From: "James B. Rawlings" <jbraw@bahaha.che.wisc.edu>  
To: John Bagterp <bagterp@bevo.che.wisc.edu>  
Date: Wed, 2 Aug 2000 07:10:34 -0500 (CDT)

p.s. If Sten doesn't get excited about the approach pretty soon, you should check his food; maybe he's on some medication.

On 3-Jul-2007, Sten Bay Jorgensen <SBJ@kt.dtu.dk> wrote:

I actually think this will be a very worthwhile contribution which indeed bridges theory and application.

## Rotated cost and Lyapunov function

- Stability results extended to nonlinear systems satisfying **strong duality**. There exists constant  $\lambda \in \mathbb{R}^n$  such that

$$\min_{(x,u) \in \mathbb{Z}} \ell(x, u) + \lambda'(x - f(x, u)) \geq \ell(x_s, u_s)$$

- The Lyapunov function is based on **rotated stage cost**

$$L(x, u) = \ell(x, u) - \ell(x_s, u_s) + \lambda'(x - f(x, u))$$

- Diehl, Amrit, and Rawlings (2011)

## Generalization to dissipative systems

- Stability results generalized to **dissipative systems**. There exists function  $\lambda : \mathbb{X} \rightarrow \mathbb{R}$  and supply rate  $s : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{R}$  such that

$$\lambda(f(x, u)) - \lambda(x) \leq s(x, u) \quad (x, u) \in \mathbb{Z}$$

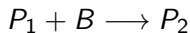
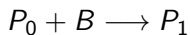
$$\min_{(x, u) \in \mathbb{Z}} \ell(x, u) + \lambda(x) - \lambda(f(x, u)) \geq \ell(x_s, u_s)$$

- Rotated stage cost

$$L(x, u) = \ell(x, u) - \ell(x_s, u_s) + \lambda(x) - \lambda(f(x, u))$$

- MPC also shown to outperform optimal periodic control
- Angeli, Amrit, and Rawlings (2011)

# Nonlinear chemical reactor example



Species mass balances:

$$\dot{x}_1 = u_1 - x_1 - \sigma_1 x_1 x_2$$

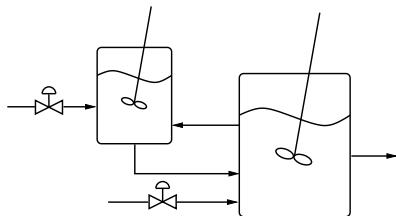
$$\dot{x}_2 = u_2 - x_2 - \sigma_1 x_1 x_2 - \sigma_2 x_2 x_3$$

$$\dot{x}_3 = -x_3 + \sigma_1 x_1 x_2 - \sigma_2 x_2 x_3$$

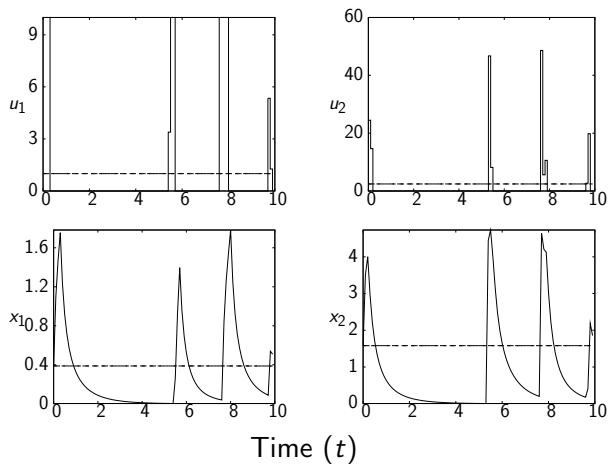
$$\dot{x}_4 = -x_4 + \sigma_2 x_2 x_3$$

$x_1, x_2, x_3, x_4$ : concentrations of  $P_0, B, P_1, P_2$

$u_1, u_2$ : inflow rates of  $P_0$  and  $B$



# Nonlinear chemical reactor example



- Optimizing economic performance may not give stability
- Tradeoff between stability and performance
- Convergence may be enforced by modifying the dynamic problem
  - ▶ Convex regularization terms in the objective

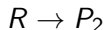
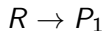
$$\tilde{\ell}(x, u) = \ell(x, u) + |x - x_s|_Q^2 + |u - u_s|_R^2$$

- ▶ Zero variance constraint

$$\text{Av}[|x - x_s|^2] \in \{0\}$$

# Enforcing convergence: Chemical reactor example

CSTR with parallel reactions



$$\dot{x}_1 = 1 - 10^4 x_1^2 e^{-1/x_3} - 400 x_1 e^{-0.55/x_3} - x_1$$

$$\dot{x}_2 = 10^4 x_1^2 e^{-1/x_3} - x_2$$

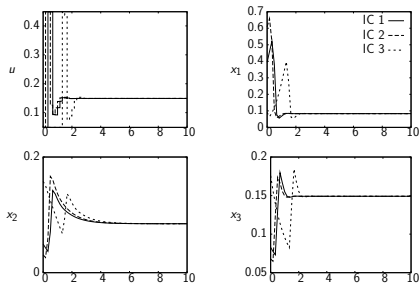
$$\dot{x}_3 = u - x_3$$

$x_1, x_2$ : Concentrations of  $R, P_1$

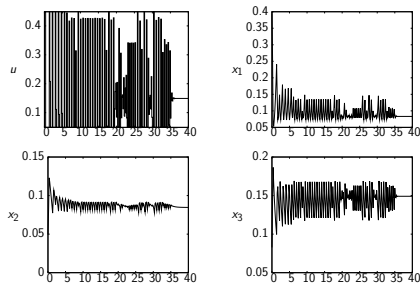
$x_3$ : Temperature in the reactor.

$u$ : Heat flux through the reactor wall

- Convex terms in objective



- Convergence constraint



- Opportunities

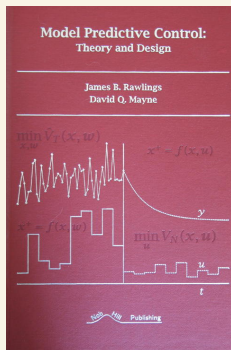
- ▶ Performance advantage
- ▶ Consistent model for optimizing performance
- ▶ Consistent statement of process objectives
- ▶ Optimization software well developed
- ▶ Linear dynamics and convex objectives well supported
- ▶ Leverage from industrial implementation of linear MPC
- ▶ This is a big opportunity!

- Challenges

- ▶ Understanding the interplay between nonlinearity of model and convexity of objective
- ▶ Managing the complexity of an optimal *economic* solution
- ▶ Developing theory, algorithms, tuning procedures, and realistic case studies that support industrial implementation
- ▶ This is a big challenge!

*Writing a book is an adventure. To begin with it is a toy, then amusement, then it becomes a mistress, then it becomes a master, and then it becomes a tyrant, and the last phase is that just as you are about to become reconciled to your servitude, you kill the monster and strew him about to the public.*

– *Winston Churchill*



- Published in 2009
- 576 page text
- 3 appendices on web (133 pages)
- 214 exercises
- 335 page solution manual
- total: 1044 pages
- [www.nobhillpublishing.com](http://www.nobhillpublishing.com)

## Some (of many!) connections between UW Madison and DTU

- Collaboration of John Villadsen (DTU) and Warren Stewart (UW) on orthogonal collocation (Villadsen and Stewart, 1967)
- Book by John Villadsen and Michael Michelsen (Villadsen and Michelsen, 1978)
- Ole Hassager (DTU); PhD (1973) in polymers with Bob Bird (UW)
- Juan de Pablo (UW); 2011 Ørsted Lectureship Award from DTU
- Manos Mavrikakis (UW) taught Catalytic and Advanced Reaction Engineering course at DTU in 2006
- Sten Bay Jørgensen and the systems area
- John Jørgensen and Dennis Bonné visited UW during their PhDs
- Emil Sokoler visited UW during his PhD and worked with me and Steve Wright, computer science. Third generation of interactions!

## Further reading I

- D. Angeli, R. Amrit, and J. B. Rawlings. On average performance and stability of economic model predictive control. *IEEE Trans. Auto. Cont.*, 2011. Accepted for publication.
- R. E. Bellman and S. E. Dreyfus. *Applied Dynamic Programming*. Princeton University Press, Princeton, New Jersey, 1962.
- A. E. Bryson and Y. Ho. *Applied Optimal Control*. Hemisphere Publishing, New York, 1975.
- M. Diehl, R. Amrit, and J. B. Rawlings. A Lyapunov function for economic optimizing model predictive control. *IEEE Trans. Auto. Cont.*, 56(3):703–707, 2011.
- R. E. Kalman. Contributions to the theory of optimal control. *Bull. Soc. Math. Mex.*, 5:102–119, 1960.
- H. Kwakernaak and R. Sivan. *Linear Optimal Control Systems*. John Wiley and Sons, New York, 1972. ISBN 0-471-51110-2.

## Further reading II

- J. B. Rawlings, D. Bonné, J. B. Jørgensen, A. N. Venkat, and S. B. Jørgensen. Unreachable setpoints in model predictive control. *IEEE Trans. Auto. Cont.*, 53 (9):2209–2215, October 2008.
- D. M. Starks and E. Arrieta. Maintaining AC&O applications, sustaining the gain. In *Proceedings of National AIChE Spring Meeting*, Houston, Texas, April 2007.
- J. Villadsen and M. L. Michelsen. *Solution of Differential Equation Models by Polynomial Approximation*. Prentice-Hall, Englewood Cliffs New Jersey, 1978.
- J. V. Villadsen and W. E. Stewart. Solution of boundary-value problems by orthogonal collocation. *Chem. Eng. Sci.*, 22:1483–1501, 1967.