

# Controlling Large-Scale Systems with Distributed Model Predictive Control

James B. Rawlings

Department of Chemical and Biological Engineering

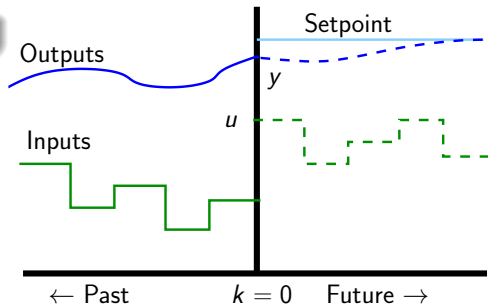


November 8, 2010  
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Salt Lake City, UT

- 1 Overview of Distributed Model Predictive Control
  - Control of large-scale systems
- 2 Cooperative Control
  - Stability theory for cooperative MPC
- 3 Conclusions and Future Outlook
- 4 Some Comments on Tom Edgar

# Model predictive control

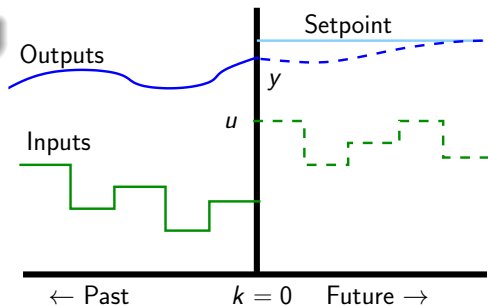
What are the goals of MPC?



# Model predictive control

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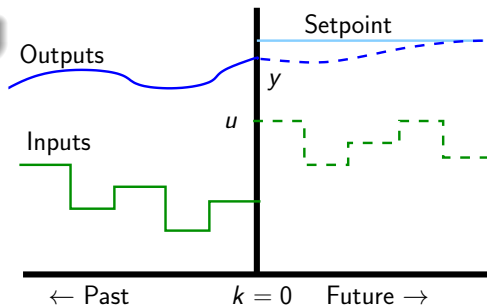
- Choose inputs which bring outputs to their setpoints



# Model predictive control

What are the goals of MPC?

- Choose inputs which bring outputs to their setpoints
- Minimize objective function over  $N$  future steps



$$\min_{\mathbf{u}} V(x, \mathbf{u}) = \sum_{k=0}^{N-1} \ell(x(k), u(k)) + V_f(x(N))$$

subject to

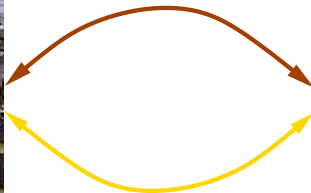
$$x^+ = Ax + Bu$$

$$y = Cx$$

# Chemical plant integration



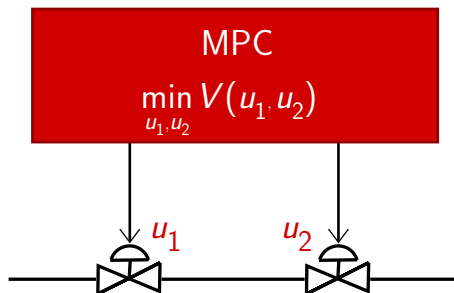
**Material flow**



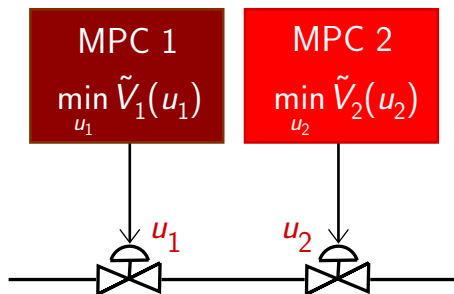
**Energy flow**



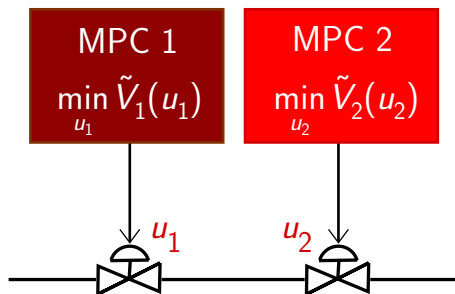
# Ideal plantwide MPC



- Ideal controller
  - ▶ perfect model
  - ▶ never goes offline
  - ▶ optimizes infinitely fast
  - ▶ samples infinitely fast

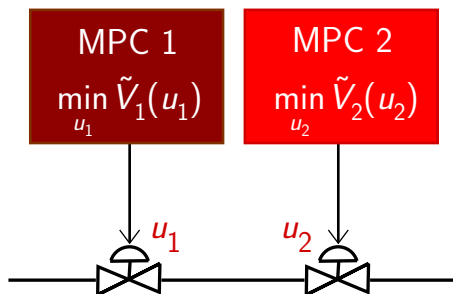


- Realistic controller
  - ▶ approximate model
  - ▶ MPCs fail or require maintenance
  - ▶ finite optimization time
  - ▶ multiple sampling rates



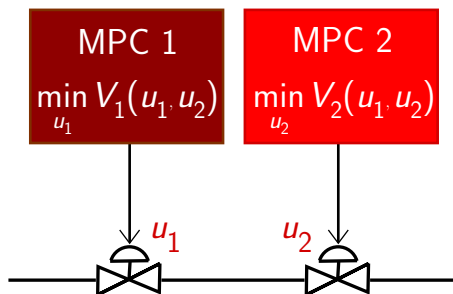
- Realistic controller
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  - ▶ multiple sampling rates
- **Goal:** make realistic controller close to ideal controller

# Plantwide distributed MPC



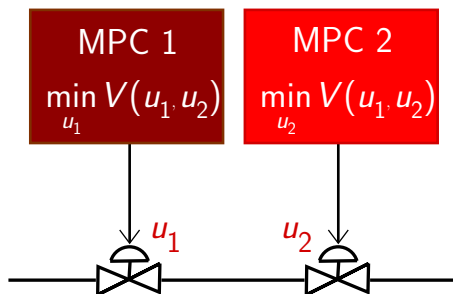
- Decentralized control
  - ▶ no communication
  - ▶ **not** stable for strongly interacting subsystems

# Plantwide distributed MPC



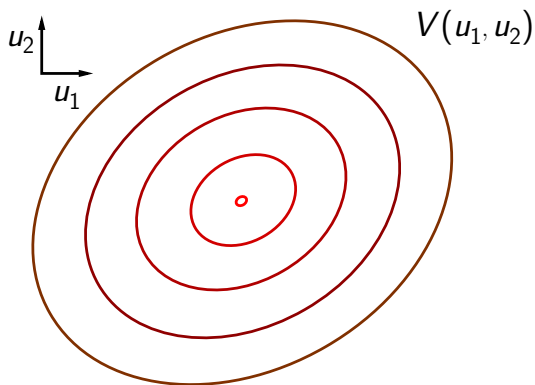
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- Cooperative control
  - ▶ use same objective in each controller
  - ▶ stability independent of interaction strength

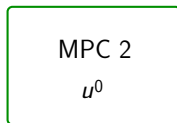
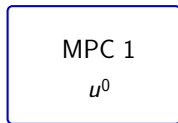
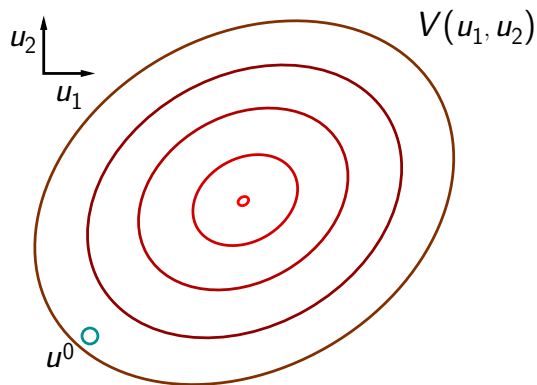
# Cooperative model predictive control



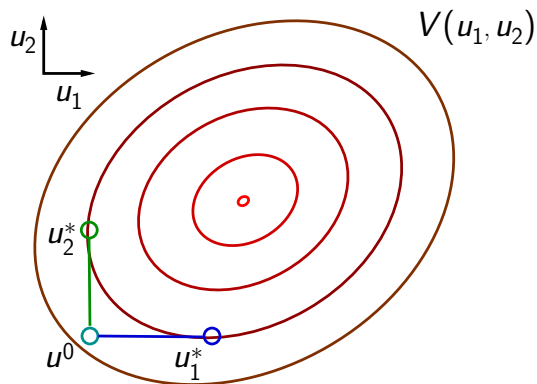
MPC 1

MPC 2

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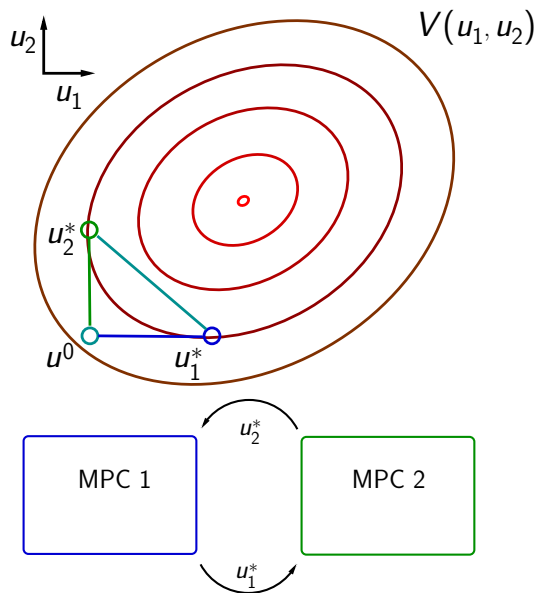
MPC 1

$u_1^*$

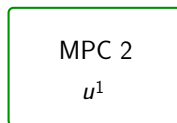
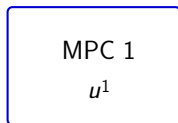
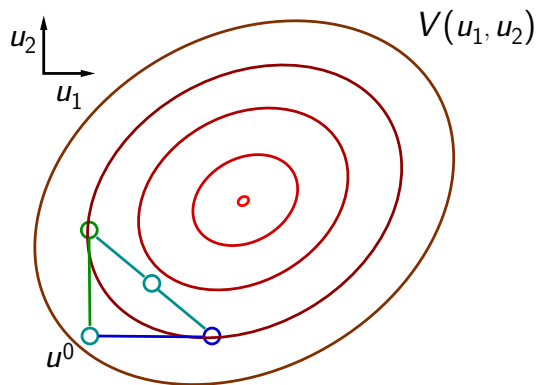
MPC 2

$u_2^*$

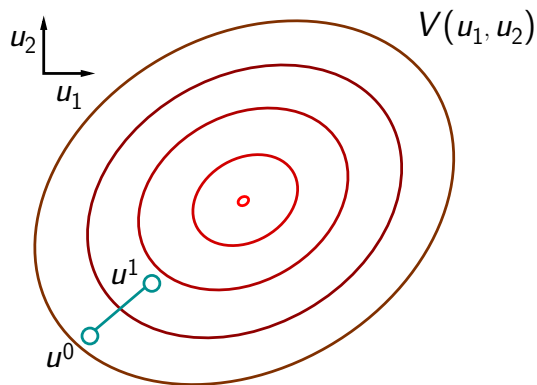
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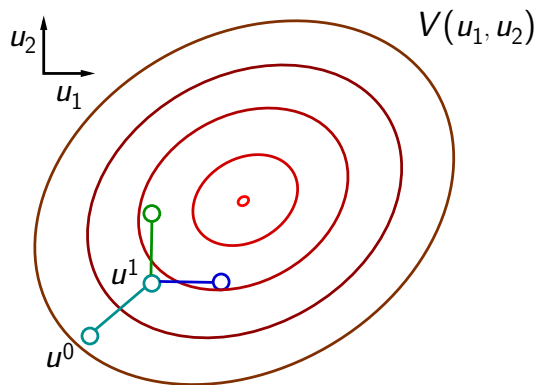
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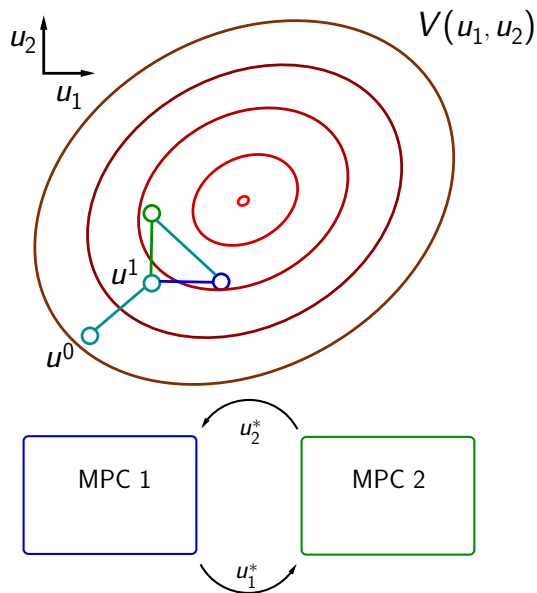
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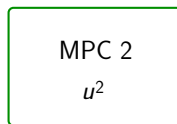
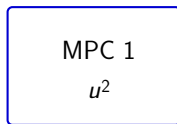
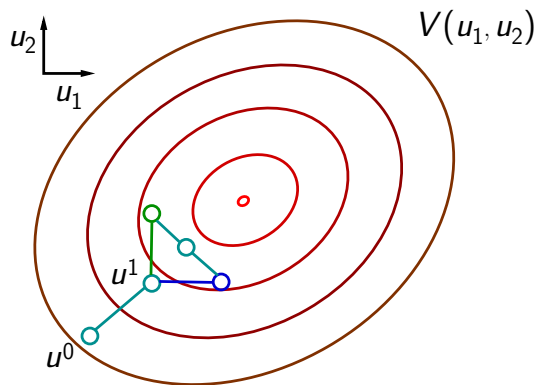
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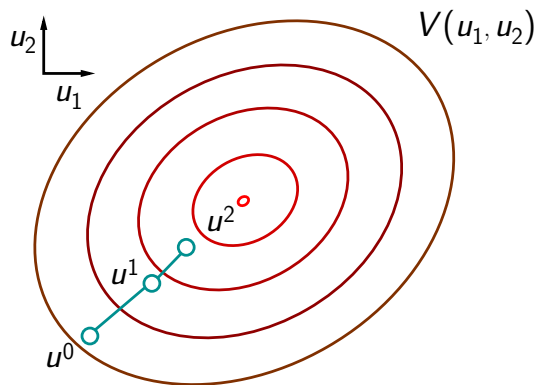
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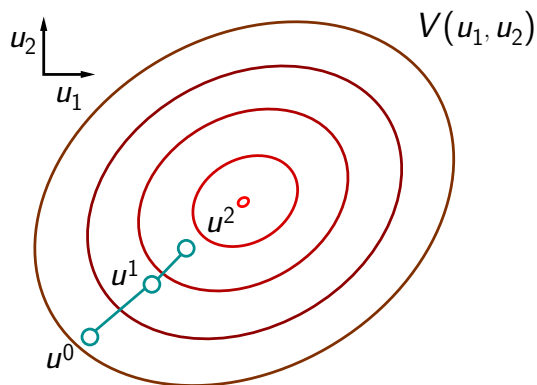
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# Plantwide suboptimal MPC



- Early termination of optimization gives suboptimal plantwide feedback
- Use suboptimal MPC theory to prove stability

## Plantwide suboptimal MPC

Consider closed-loop system augmented with input trajectory

$$\begin{pmatrix} x^+ \\ \mathbf{u}^+ \end{pmatrix} = \begin{pmatrix} Ax + Bu \\ g(x, \mathbf{u}) \end{pmatrix}$$

- Function  $g(\cdot)$  returns suboptimal choice

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- Stability of augmented system is established by Lyapunov function

$$\begin{aligned} a |(x, \mathbf{u})|^2 &\leq V(x, \mathbf{u}) \leq b |(x, \mathbf{u})|^2 \\ V(x^+, \mathbf{u}^+) - V(x, \mathbf{u}) &\leq -c |(x, \mathbf{u})|^2 \end{aligned}$$

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- Adding constraint establishes closed-loop stability of the origin for all  $\mathbf{u}^1$

$$|\mathbf{u}| \leq d |x| \quad x \in \mathbb{B}_r, r > 0$$

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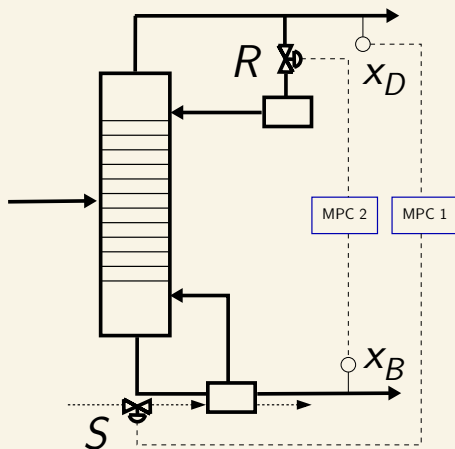
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- Cooperative optimization satisfies these properties for plantwide objective function  $V(x, \mathbf{u})$

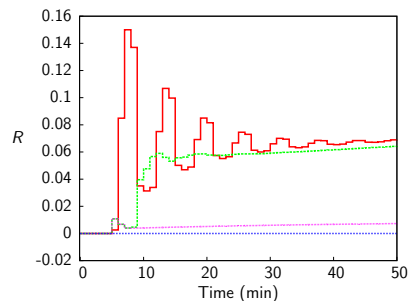
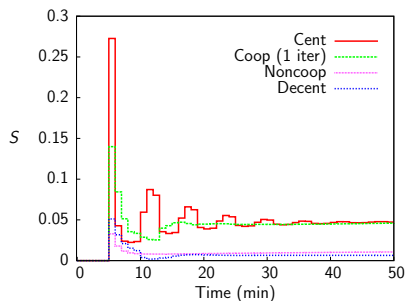
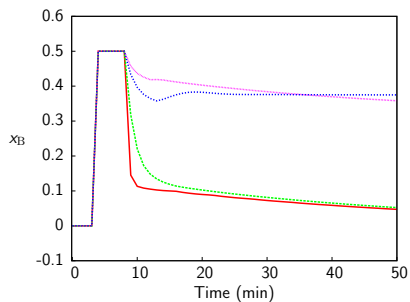
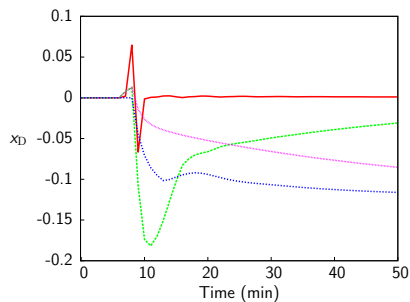
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# LV control of distillation column

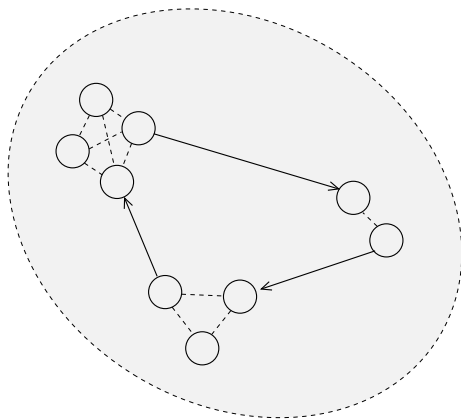


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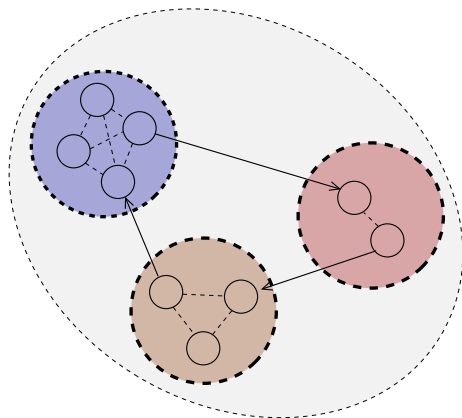
## Performance comparison

	Cost	Performance loss (%)
Centralized MPC	75.8	0
Cooperative MPC (10 iterates)	76.1	0.388
Cooperative MPC (1 iterate)	87.5	15.4
Noncooperative MPC	382	404
Decentralized MPC	364	380



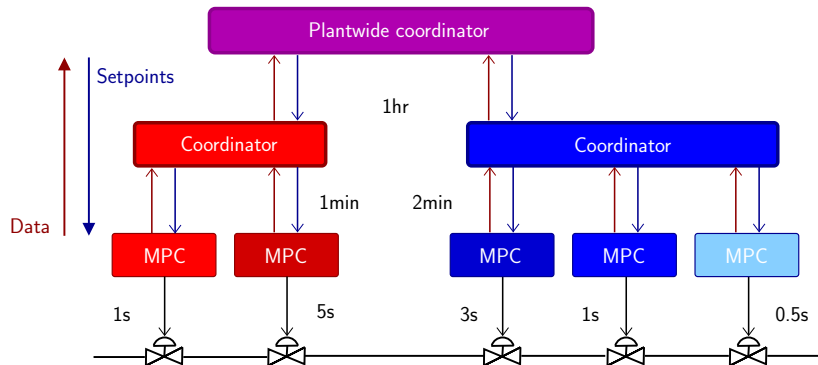
- Plant subsystems can often be grouped spatially or dynamically

# Plantwide topology



- Plant subsystems can often be grouped spatially or dynamically
- Neighborhoods of subsystems naturally arise from topology

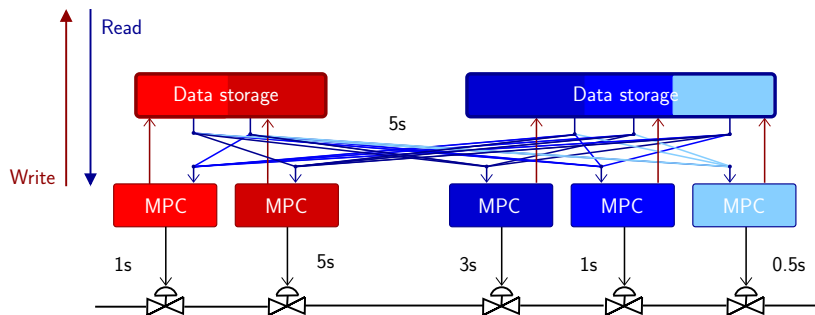
# Traditional hierarchical MPC<sup>2</sup>



- Multiple dynamical time scales in plant
- Data and setpoints are exchanged on chosen scale
- Optimization performed at each layer

<sup>2</sup>Mesarović et al. (1970); Scattolini (2009)

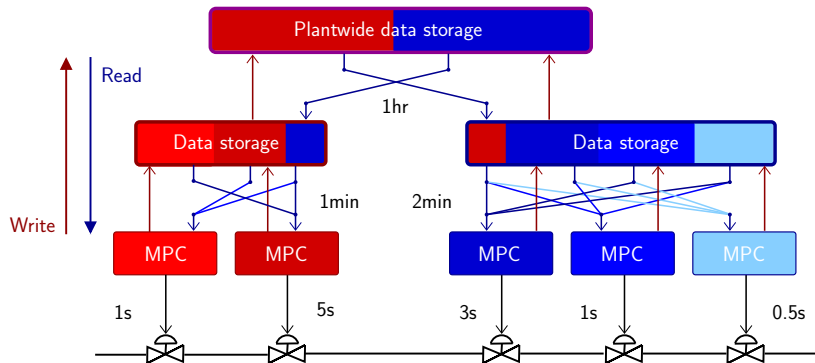
# Cooperative MPC data exchange<sup>3</sup>



- All data exchanged plantwide
- Data exchange at each controller execution

<sup>3</sup>Venkat (2006); Stewart et al. (2010b)

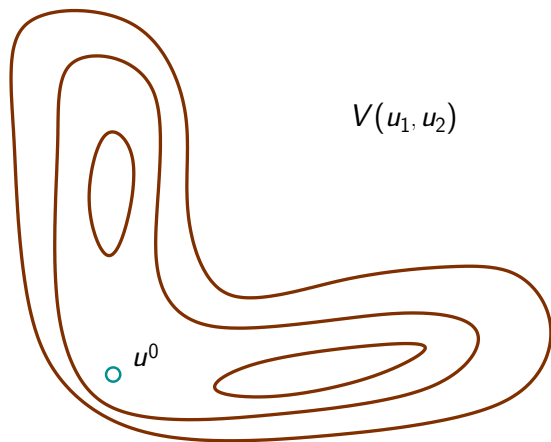
# Cooperative hierarchical MPC<sup>4</sup>



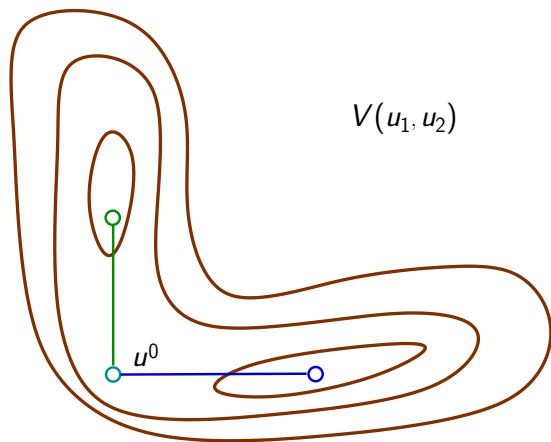
- Optimization at MPC layer only
- Only subset of data exchanged plantwide
- Data exchanged at chosen time scale

<sup>4</sup>Stewart et al. (2010a)

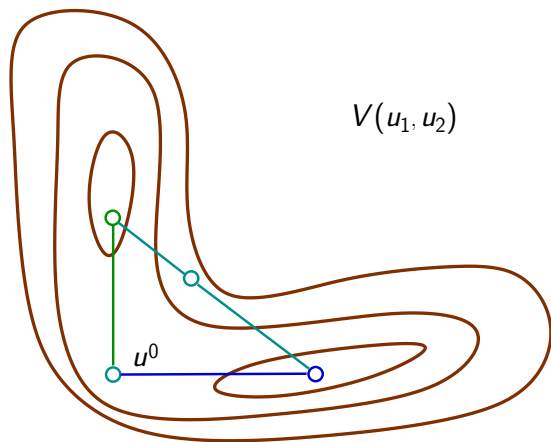
# The challenge of nonlinear models



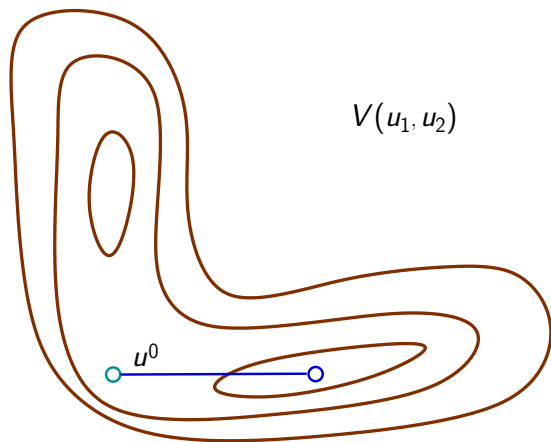
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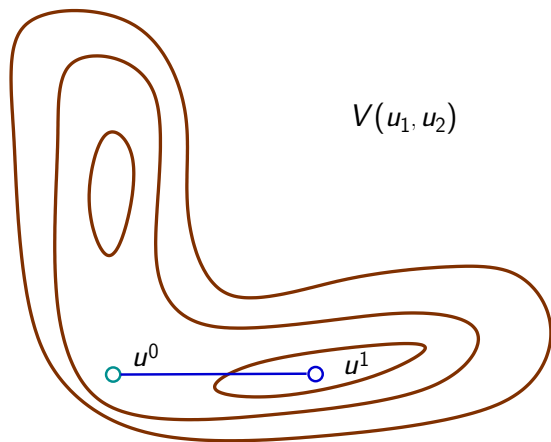
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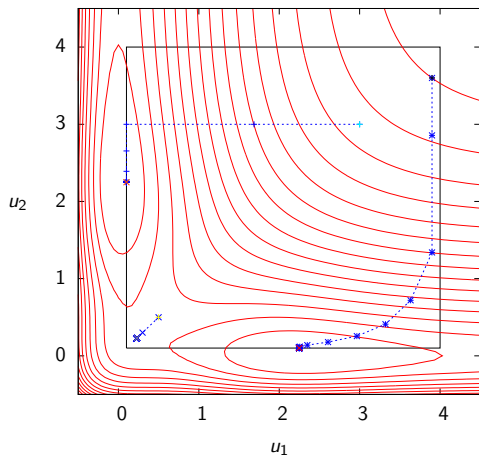
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## Distributed gradient projection - example



$$V(u_1, u_2) = e^{-2u_1} - 2e^{-u_1} + e^{-2u_2} - 2e^{-u_2} \\ + 1.1 \exp(-0.4((u_1 + 0.2)^2 + (u_2 + 0.2)^2))$$

Cooperative MPC theory maturing (Stewart et al., 2010b; Maestre et al., 2010)

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- Retains closed-loop stability for early iteration termination
- Converges to Pareto optimal control in the limit of iteration
- Remains stable under perturbation from stable state estimator
- Avoids coordination layer

Extensions required for practical implementation

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- He's a lot of fun to be around

## Further reading I

- J. M. Maestre, D. Muñoz de la Peña, and E. F. Camacho. Distributed model predictive control based on a cooperative game. *Optimal Cont. Appl. Meth.*, In press, 2010.
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