Optimal control unchained

James B. Rawlings

Department of Chemical and Biological Engineering

THE UNIVERSITY of WISCONSIN MADISON

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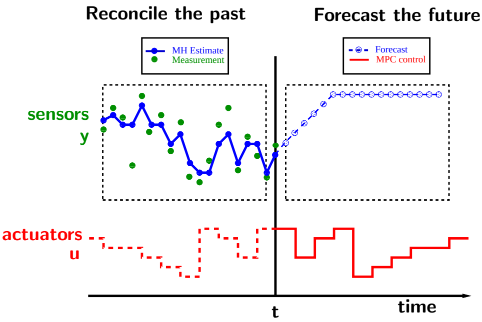
Outline

1. Optimal control, optimal feedback control, and model predictive control (MPC)
2. Industrial impact of these ideas
3. Are all the problems solved?
   - Distributed control for large-scale systems
   - Optimizing economics
4. Conclusions and future outlook

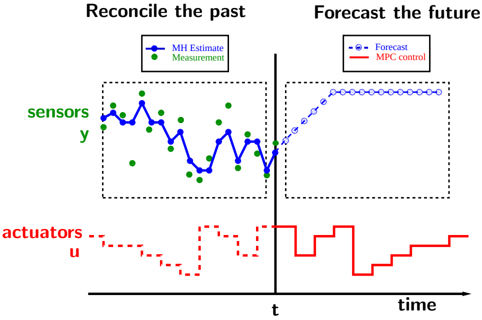
Predictive control



$$\min_{u(t)} \int_0^T |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2 \, dt$$

$$\dot{x} = f(x,u)$$
$$x(0) = x_0 \quad \text{(given)}$$
$$y = g(x,u)$$

State estimation



$$\min_{x_0, w(t)} \int_{-T}^{0} |y - g(x,u)|_R^2 + |\dot{x} - f(x,u)|_Q^2 \, dt$$

$$\begin{aligned} \dot{x} &= f(x,u) + w \quad &\text{(process noise)} \\ y &= g(x,u) + v \quad &\text{(measurement noise)} \end{aligned}$$

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- Optimal feedback control sees limited industrial application during this period.

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Our notion of very rapidly changed radically from 1960 to 1985.

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Linear MPC and ethylene manufacturing

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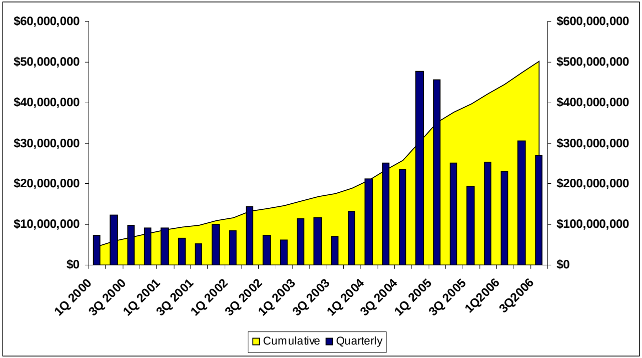
Dow Hydrocarbons AC&O 17

We're Doing it For the Money



Advanced Control & Optimization

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Some questions to consider

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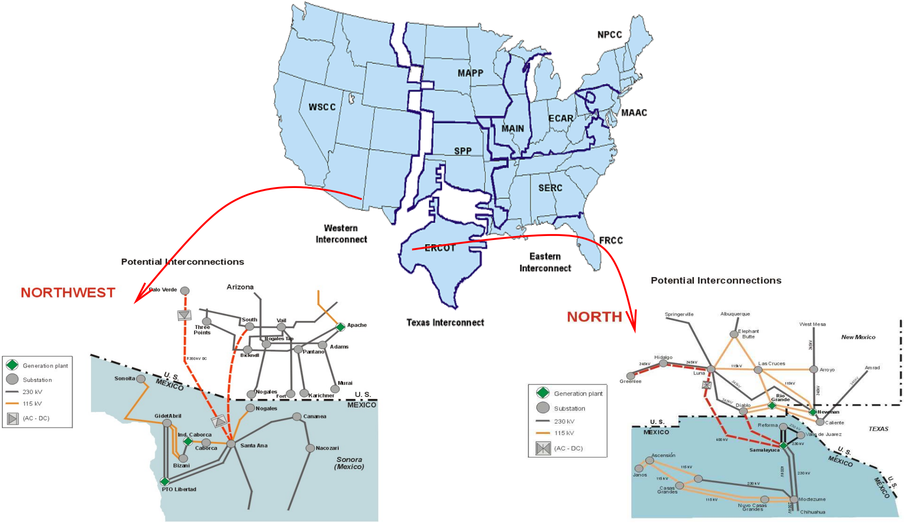
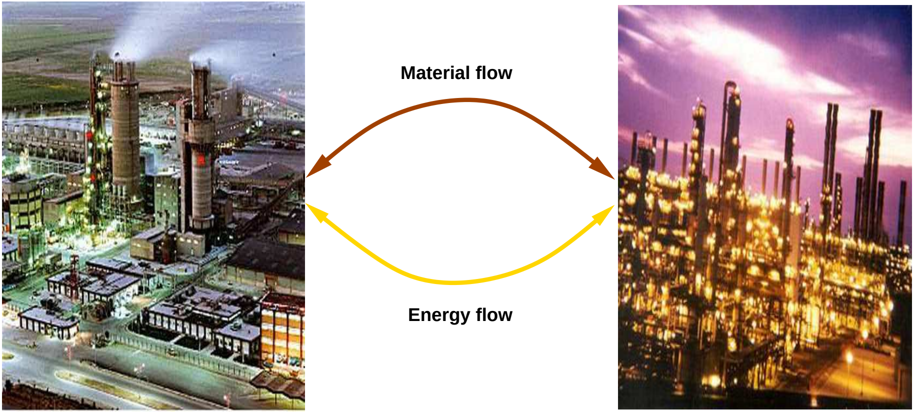
- Do we have tools to decompose *large-scale systems* into manageable problems?
- Do we have tools to *commission* and *maintain* the controllers?
- Do we have tools to optimize dynamic *economic* operation?

Distributed MPC — Decomposing large-scale systems



Decomposing large-scale systems



Game theory — theoretical framework for distributed MPC

Objective functions	$V_1(u_1, u_2),\ \ V_2(u_1, u_2)$
and	$V(u_1, u_2) = w_1 V_1(u_1, u_2) + w_2 V_2(u_1, u_2)$
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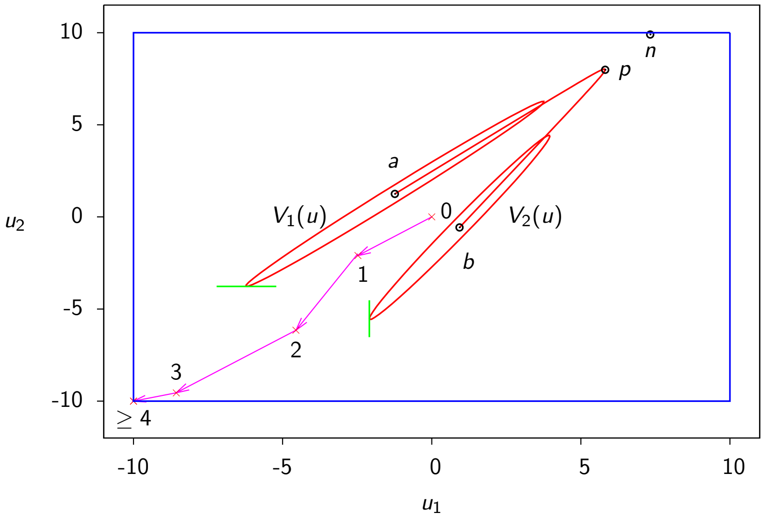
Centralized Control	$\min_{u_1, u_2 \in \Omega_1 \times \Omega_2} V(u_1, u_2)$
(Pareto optimal)	

von Neumann and Morgenstern (1944), Nash (1951), and Başar and Olsder (1999)

Geometry of cooperative vs. noncooperative MPC



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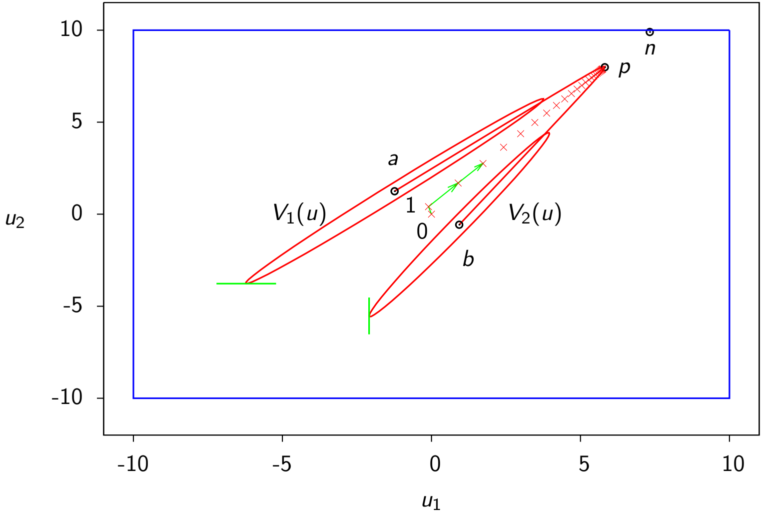

Cooperative control as suboptimal MPC

Properties established by suboptimal MPC theory[1]

- Stability: Nominal closed-loop stability for *any number* of information exchanges

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- Cost decrease: Plant-wide objective is decreased at each iterate
- Convergence: Cooperative MPC produces centralized control performance at the limit of infinite iterates

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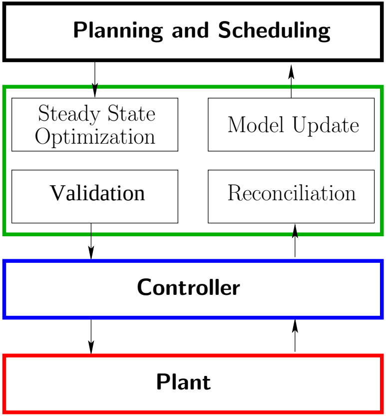
Optimizing economics: Current industrial practice



- Two layer structure
- Drawbacks

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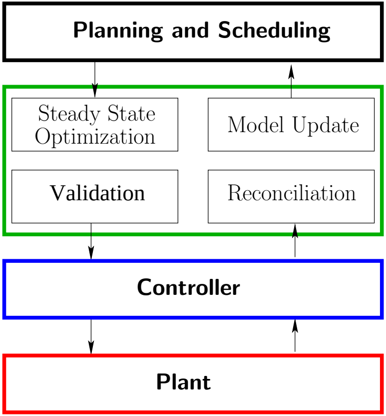
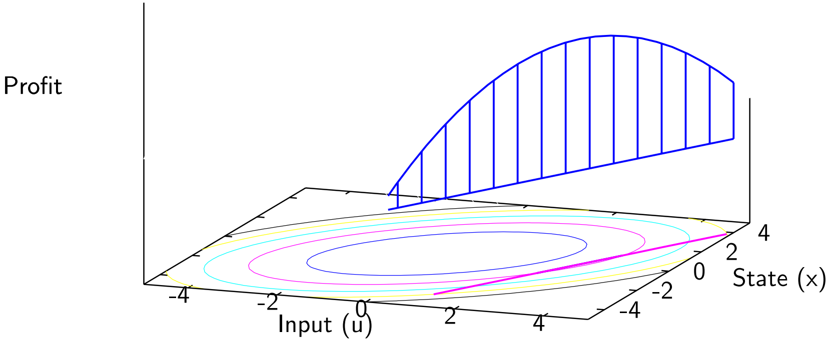
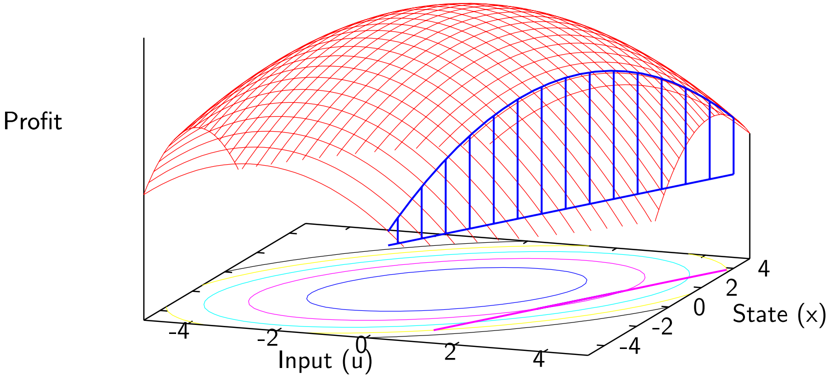


- Two layer structure
- Drawbacks
  - Inconsistent models
  - Re-identify linear model as setpoint changes
  - Time scale separation may not hold
  - Economics unavailable in dynamic layer

Optimizing economics: what's desirable?



Optimizing economics: what's desirable?



Recent results for economic MPC

- The economic MPC for linear dynamics and convex cost is asymptotically stabilizing
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- A Lyapunov function has recently been found for this case (Diehl et al., 2009)
- Both the theory and the computation for the nonlinear model and nonconvex cost require further work (Würth et al., 2009; Angeli et al., 2009)

Conclusions

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- The currently available theory splits the problem into state estimation and regulation. Both are posed and solved as online optimization problems. Basic properties have been established.
- The economic impact of the technology is large.

Future outlook

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2. Optimizing dynamic economic objective. Theory available for linear models and convex cost functions. Extensions to nonlinear models and nonconvex cost functions at early stages.

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